2011

10. (a) If

$$x^2\frac{dy}{dx} - xy = 7y$$

and $y = 1$ when $x = 1$, find the value of y when $x = 2$.

$$x^2\frac{dy}{dx} = xy + 7y$$

$$\frac{dy}{dx} = \frac{y(x+7)}{x^2}$$

$$\int\frac{1}{y}dy = \int\frac{x+7}{x^2}\,dx \qquad 5$$

$$\int\frac{1}{y}dy = \int\left(\frac{1}{x} + \frac{7}{x^2}\right)dx$$

$$\ln y = \ln x - \frac{7}{x} + C \qquad 5$$

$$y = 1,\ x = 1$$

$$\Rightarrow C = 7 \qquad 5$$

$$\ln y = \ln x - \frac{7}{x} + 7$$

$$\ln y = \ln 2 - \frac{7}{2} + 7$$

$$= 4.1931$$

$$\Rightarrow y = e^{4.1931} = 66.23 \qquad 5 \qquad 20$$