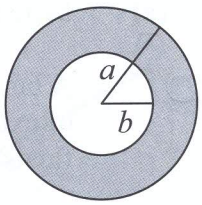
8. (b) An annulus is created when a central hole of radius b is removed from a uniform circular disc of radius a.

2010



The mass of the annulus (shaded area) is M.

(i) Show that the moment of inertia of the annulus about an axis through its centre and perpendicular to its plane is $\dfrac{M(a^2+b^2)}{2}$.

(ii) The annulus rolls, from rest, down an incline of 30°. Find its angular velocity, in terms of g, a and b, when it has rolled a distance $\dfrac{a}{2}$.

(i) moment of inertia of annulus $= 2\pi M_1 \int_b^a x^3\,dx$ — 5

$= 2\pi M_1 \left[\dfrac{x^4}{4}\right]_b^a$

$= 2\pi \dfrac{M}{\pi(a^2-b^2)} \dfrac{(a^4-b^4)}{4}$ — 5

$= \dfrac{M(a^2+b^2)}{2}$ — 5

(ii) Gain in KE $=$ Loss in PE

$\frac{1}{2} I\omega^2 + \frac{1}{2} Mv^2 = Mgh$ — 5

$\frac{1}{2} I\omega^2 + \frac{1}{2} M(a\omega)^2 = Mg\left(\dfrac{a}{2}\sin 30\right)$ — 5

$\dfrac{1}{2}\left\{\dfrac{M(a^2+b^2)}{2}\right\}\omega^2 + \frac{1}{2} M(a\omega)^2 = Mg\left(\dfrac{a}{4}\right)$

$\omega = \sqrt{\dfrac{ga}{3a^2+b^2}}$ — 5

30