

3 (b)

2010

A particle is projected up an inclined plane with initial speed 80 m s^{-1} . The line of projection makes an angle of 30° with the inclined plane and the plane is inclined at an angle θ to the horizontal. The plane of projection is vertical and contains the line of greatest slope.

The particle strikes the plane at an angle of $\tan^{-1} \frac{2}{\sqrt{3}}$.

Find (i) the value of θ

(ii) the speed with which the particle strikes the plane.

(i)

$$r_j = 0$$

$$0 = 80 \sin 30^\circ t - \frac{1}{2} g \cos \theta t^2$$

$$\Rightarrow t = \frac{80}{g \cos \theta}$$

$$\begin{aligned} v_i &= 80 \cos 30^\circ - g \sin \theta \left(\frac{80}{g \cos \theta} \right) \\ &= 40\sqrt{3} - 80 \tan \theta \end{aligned}$$

$$\begin{aligned} v_j &= 80 \sin 30^\circ - g \cos \theta \left(\frac{80}{g \cos \theta} \right) \\ &= -40 \end{aligned}$$

$$\tan \ell = \frac{-v_j}{v_i}$$

$$\frac{2}{\sqrt{3}} = \frac{40}{40\sqrt{3} - 80 \tan \theta}$$

$$\tan \theta = \frac{\sqrt{3}}{4} \Rightarrow \theta = 23.4^\circ$$

(ii)

$$v_i = 20\sqrt{3}$$

$$v_j = -40$$

$$\begin{aligned} \text{speed} &= \sqrt{(20\sqrt{3})^2 + (-40)^2} \\ &= 20\sqrt{7} \text{ or } 52.9 \text{ m s}^{-1} \end{aligned}$$

5

5

5

5

5

5

30