10. (a) Solve the differential equation

$$y\frac{dy}{dx} = x + xy^2$$

2010

given that $y = 0$ when $x = 0$.

$$y\frac{dy}{dx} = x + xy^2$$

$$\frac{dy}{dx} = \frac{x(1+y^2)}{y}$$

$$\int \frac{y}{1+y^2}\,dy = \int x\,dx \qquad 5$$

$$\frac{1}{2}\ln(1+y^2) = \frac{1}{2}x^2 + C \qquad 5$$

$$y = 0,\ x = 0$$

$$\Rightarrow C = 0 \qquad 5$$

$$\frac{1}{2}\ln(1+y^2) = \frac{1}{2}x^2$$

$$1+y^2 = e^{x^2}$$

$$\Rightarrow y = \sqrt{e^{x^2} - 1} \qquad 5$$

20