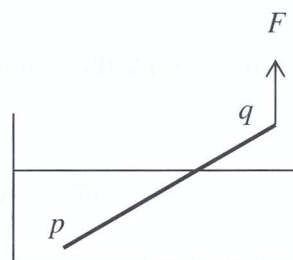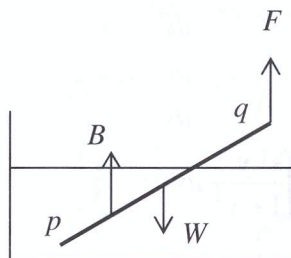**9** **(b)** A thin uniform rod pq of weight W is in equilibrium in an inclined position with end p immersed in a container of water. The end q is supported by a vertical force F, as shown in the diagram. The relative density of the material of the rod is s.

(i) Find in terms of s the fraction of the length of the rod that is immersed.

(ii) If $s = \dfrac{3}{4}$, find F in terms of W.

(i) Let length of immersed part $= x$

Take moments about q:

$$B\left(\ell - \frac{x}{2}\right)\sin\theta = W\tfrac{1}{2}\ell\sin\theta$$

5,5

$$B = \frac{\frac{x}{\ell}W(1)}{s} = \frac{xW}{\ell s}$$

5

$$\left(\frac{xW}{\ell s}\right)\left(\ell - \frac{x}{2}\right)\sin\theta = W\tfrac{1}{2}\ell\sin\theta$$

$$x^2 - 2\ell x + \ell^2 s = 0$$

$$x = \frac{2l \pm \sqrt{4\ell^2 - 4\ell^2 s}}{2}$$

$$\text{fraction} = \frac{x}{\ell} = 1 - \sqrt{1 - s}$$

5

(ii)

$$\frac{x}{\ell} = 1 - \sqrt{1 - \tfrac{3}{4}} = \frac{1}{2}$$

$$\Rightarrow B = \frac{W}{2s} = \frac{2W}{3}$$

$$B + F = W$$

$$\frac{2W}{3} + F = W$$

$$\Rightarrow F = \frac{W}{3}$$

5   25