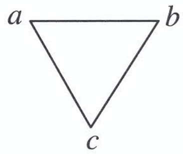
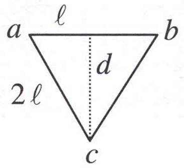
8 **(b)** Three equal uniform rods, each of length 2ℓ and mass m, form the sides of an equilateral triangle abc.

(i) Find the moment of inertia of the frame abc about an axis through a perpendicular to the plane of the triangle.

The triangular frame abc is attached to a smooth hinge at a about which it can rotate in a vertical plane. The frame is held with ab horizontal, and c below ab, and then released from rest.



(ii) Find the maximum angular velocity of the triangle in the subsequent motion.



(i)

$$(2\ell)^2 = \ell^2 + d^2$$

$$d = \ell\sqrt{3}$$ 5

$$I = \tfrac{4}{3}m\ell^2 + \tfrac{4}{3}m\ell^2 + \left\{\tfrac{1}{3}m\ell^2 + md^2\right\}$$

$$= 3m\ell^2 + md^2$$

$$= 3m\ell^2 + m\left(\ell\sqrt{3}\right)^2$$

$$= 6m\ell^2$$ 5

(ii) Gain in KE = Loss in PE

$$\tfrac{1}{2}I\omega^2 = Mgh$$

$$= 3mgh$$ 5

$$h = \tfrac{1}{3}d$$ 5

$$\tfrac{1}{2}\left(6m\ell^2\right)\omega^2 = (3m)g\left(\tfrac{1}{3}d\right)$$ 5

$$3\ell^2\omega^2 = g\left(\ell\sqrt{3}\right)$$

$$\omega = \sqrt{\frac{g\sqrt{3}}{3\ell}}$$ 5 | 30