8. **(a)** Prove that the moment of inertia of a uniform rod of mass m and length 2ℓ about an axis through its centre perpendicular to the rod is $\frac{1}{3}m\ell^2$.

Let M = mass per unit length

mass of element $= M\{dx\}$

moment of inertia of the element $= M\{dx\}x^2$ — 5

moment of inertia of the rod $= M\int_{-\ell}^{\ell} x^2\,dx$ — 5

$= M\left[\frac{x^3}{3}\right]_{-\ell}^{\ell}$ — 5

$= \frac{2}{3}M\ell^3$

$= \frac{1}{3}m\ell^2$ — 5 | 20