7   **(b)**   A uniform rod of length $2p$ and weight W rests with
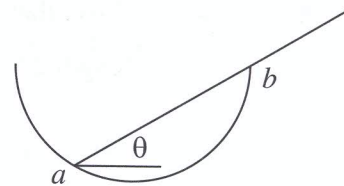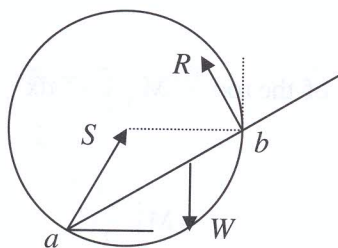its lower end a in contact with a smooth
hemispherical bowl, of radius p.
The axis of the bowl is vertical.

The upper end of the rod projects beyond
the rim of the bowl as shown in the diagram.
The inclination of the rod to the horizontal is θ.
The point b on the rod is in contact with the rim of the bowl.
$|ab| = 2p\cos\theta$.

(i)   Find, in terms of W, the reaction at b.

(ii)   Show that $\cos\theta = 2\cos 2\theta$.

(i)   Moments about a :

$$R(2p\cos\theta) = W(p\cos\theta)$$

$$R = \frac{W}{2}$$

5

5

(ii) $\alpha = \theta$

5

horiz   : $R\sin\theta = S\cos 2\theta$

$$\Rightarrow S = \frac{R\sin\theta}{\cos 2\theta} = \frac{W\sin\theta}{2\cos 2\theta}$$

vert   : $R\cos\theta + S\sin 2\theta = W$

5

$$\left(\frac{W}{2}\right)\cos\theta + \left(\frac{W\sin\theta}{2\cos 2\theta}\right)\sin 2\theta = W$$

$$\cos 2\theta\cos\theta + \sin 2\theta\sin\theta = 2\cos 2\theta$$

$$\cos(2\theta - \theta) = 2\cos 2\theta$$

$$\cos\theta = 2\cos 2\theta$$

5   25