6. **(a)** The distance, x, of a particle from a fixed point, o, is given by

$$x = a\cos(\omega t + \varepsilon)$$

where a, ω and ε are constants.

(i) Show that the motion of the particle is simple harmonic.

A particle moving with simple harmonic motion starts from a point 5 cm from the centre of the motion with a speed of 1 cm/s.

(ii) The period of the motion is 11 seconds. Find the maximum speed of the particle, correct to two decimal places.

(i)

$$x = a\cos(\omega t + \varepsilon)$$
$$\dot{x} = -a\omega\sin(\omega t + \varepsilon)$$
$$\ddot{x} = -a\omega^2\cos(\omega t + \varepsilon)$$
$$= -\omega^2 x$$
$$\Rightarrow \text{S.H.M. about } x = 0.$$

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5

(ii) Period $= 11$

$$\frac{2\pi}{\omega} = 11$$
$$\omega = \frac{2\pi}{11} \text{ or } \frac{4}{7}$$

5

$x = 5,\ t = 0 \Rightarrow 5 = a\cos\varepsilon$

$v = 1,\ t = 0 \Rightarrow 1 = -a\omega\sin\varepsilon$

$$\cos\varepsilon = \frac{5}{a} \Rightarrow \sin\varepsilon = \frac{\sqrt{a^2 - 25}}{a}$$
$$\Rightarrow 1 = -a\left(\frac{4}{7}\right)\frac{\sqrt{a^2 - 25}}{a}$$
$$a = 5.3$$

$$v = \omega\sqrt{a^2 - x^2}$$
$$1 = \frac{4}{7}\sqrt{a^2 - 25}$$
$$a = 5.3$$

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$$v_{\max} = \omega a = \frac{4}{7} \times 5.3$$
$$= 3.03 \text{ cm/s.}$$

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25