1. (b) A train accelerates uniformly from rest to a speed v m/s with uniform acceleration f m/s^2. It then decelerates uniformly to rest with uniform retardation $2f$ m/s^2. The total distance travelled is d metres.

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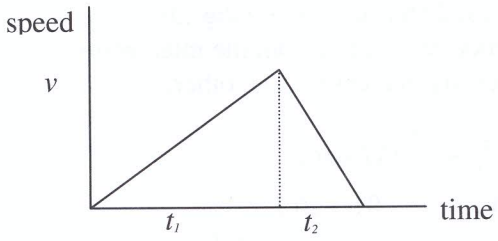
(i) Draw a speed-time graph for the motion of the train.



(ii) If the average speed of the train for the wh ole journey is $\sqrt{\frac{d}{3}}$, find the value of f.

$$f = \tan\alpha = \frac{v}{t_1} \Rightarrow t_1 = \frac{v}{f}$$

$$2f = \tan\beta = \frac{v}{t_2} \Rightarrow t_2 = \frac{v}{2f}$$

total time $\quad t_1 + t_2 = \frac{v}{f} + \frac{v}{2f} = \frac{3v}{2f}$

total distance $\quad d = \frac{1}{2}(t_1 + t_2)v$ or $t_1 + t_2 = \frac{2d}{v}$

$$t_1 + t_2 = \frac{2d}{v} \Rightarrow \frac{3v}{2f} = \frac{2d}{v} \Rightarrow 3v^2 = 4fd$$

$$\text{average speed} = \frac{\text{total distance}}{\text{total time}}$$

$$\sqrt{\frac{d}{3}} = \frac{d}{t_1 + t_2}$$

$$\sqrt{\frac{d}{3}} = \frac{d}{\frac{3v}{2f}} = \frac{2fd}{3v} \quad \text{or} \quad \sqrt{\frac{d}{3}} = \frac{v}{2}$$

$$\frac{d}{3} = \frac{4f^2d^2}{9v^2} \quad \text{or} \quad \frac{d}{3} = \frac{v^2}{4}$$

$$3v^2 = 4f^2d \quad \text{or} \quad 3v^2 = 4d$$

$$4fd = 4f^2d \quad \text{or} \quad 4fd = 4d$$

$$\Rightarrow f = 1$$

5,5 | 5 | 5 | 5 | 5 | 30