10. **(a)** Solve the differential equation

$$\frac{dy}{dx} = \frac{1}{xy} + \frac{y}{x}$$

given that $y = \sqrt{3}$ when $x = 1$.

$$\frac{dy}{dx} = \frac{1}{xy} + \frac{y}{x}$$

$$\frac{dy}{dx} = \frac{1+y^2}{xy} \qquad 5$$

$$\int \frac{y}{1+y^2}\,dy = \int \frac{1}{x}\,dx \qquad 5$$

$$\frac{1}{2}\ln\left(1+y^2\right) = \ln x + C \qquad 5$$

$$y = \sqrt{3},\ x = 1$$

$$\Rightarrow C = \frac{1}{2}\ln 4 \text{ or } \ln 2 \qquad 5$$

$$\frac{1}{2}\ln\left(1+y^2\right) = \ln x + \ln 2$$

$$\ln\left(1+y^2\right)^{\frac{1}{2}} = \ln 2x$$

$$\Rightarrow 1+y^2 = 4x^2$$

$$\Rightarrow y = \sqrt{4x^2-1} \qquad 5$$

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