

2008 10

- (b) A train of mass 200 tonnes moves along a straight level track against a resistance of  $400v^2$ , where  $v$  m/s is the speed of the train. The engine exerts a constant power of  $P$  kW.

The acceleration of the train is  $\frac{8000 - v^3}{500v}$ .

- (i) Find the value of  $P$ .  
 (ii) The train travels a distance 69.07 m while its speed increases from 10 m/s to  $v_1$  m/s. Find the value of  $v_1$ .

$$(i) \quad T = \frac{1000P}{v}$$

$$\text{Force} = \text{Mass} \times \text{Acceleration}$$

$$\frac{1000P}{v} - 400v^2 = 200\,000 \left( \frac{8000 - v^3}{500v} \right)$$

$$1000P - 400v^3 = 3200\,000 - 400v^3$$

$$P = 3200$$

$$(ii) \quad v \frac{dv}{ds} = \frac{8000 - v^3}{500v}$$

$$\int_{10}^{v_1} \frac{500v^2}{8000 - v^3} dv = \int_0^{69.07} dx$$

$$\left[ -\frac{500}{3} \ln(8000 - v^3) \right]_{10}^{v_1} = [x]_0^{69.07}$$

$$-\frac{500}{3} \ln(8000 - v_1^3) + \frac{500}{3} \ln(7000) = 69.07$$

$$\frac{500}{3} \ln \left( \frac{7000}{8000 - v_1^3} \right) = 69.07$$

$$v_1^3 = 3374.936276$$

$$v_1 = 15.0 \text{ m/s}$$