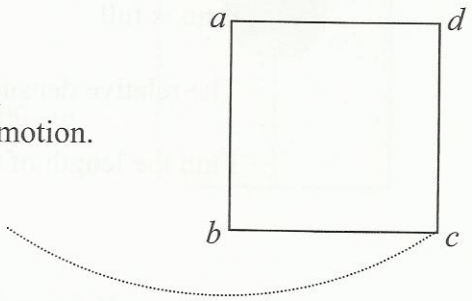
2007 8

(b) **(i)** A uniform square lamina *abcd* of side $2r$ oscillates in its own plane about a horizontal axis through a, perpendicular to its plane.

If the period of small oscillations is $2\pi\sqrt{\frac{8}{3g}}$, find the value of r.

(ii) If the lamina is released from rest when ab is vertical, find the maximum velocity of corner c in the subsequent motion.



(i)

$$I = \frac{4}{3}(m)r^2 + \frac{4}{3}(m)r^2 \qquad 5$$

$$= \frac{8}{3}(m)r^2$$

$$Mgh = mgr\sqrt{2} \qquad 5$$

$$T = 2\pi\sqrt{\frac{I}{Mgh}} \qquad 5$$

$$= 2\pi\sqrt{\frac{\frac{8}{3}(m)r^2}{mgr\sqrt{2}}}$$

$$= 2\pi\sqrt{\frac{8r}{3g\sqrt{2}}}$$

$$\Rightarrow r = \sqrt{2} \qquad 5$$

(ii)

$$\text{Gain in KE} = \text{Loss in PE}$$

$$\tfrac{1}{2}I\omega^2 = mgh$$

$$\frac{1}{2}\left(\frac{8}{3}(m)r^2\right)\omega^2 = mg(r\sqrt{2} - r) \qquad 5$$

$$\Rightarrow \omega = 1.467$$

$$\Rightarrow v = 4\omega = 5.87 \text{ m/s} \qquad 5$$

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