

2007 8.

- (a) Prove that the moment of inertia of a uniform square lamina, of mass m and side $2r$, about an axis through its centre parallel to one of the sides is $\frac{1}{3}mr^2$.

Let M = mass per unit area

$$\text{mass of element} = M\{2r \, dx\}$$

$$\text{moment of inertia of the element} = M\{2r \, dx\}x^2$$

$$\text{moment of inertia of the lamina} = M 2r \int_{-r}^r x^2 \, dx$$

$$= M 2r \left[\frac{x^3}{3} \right]_{-r}^r$$

$$= \frac{4}{3} M r^4$$

$$= \frac{1}{3} m r^2$$

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