~~1998~~ 1999

8 (b) A uniform rod of mass m is free to rotate in a vertical plane about an axis which is perpendicular to the rod and 0.32 m from its centre of gravity. For small oscillations the rod has the same period as a simple pendulum of length 0.5 m.

(i) Find the length of the rod.

(ii) For what other distance between the axis and the centre of gravity will the period be the same?

(iii) Where must the axis be located to give a minimum period?

(i)

$$I = \tfrac{1}{3} m \ell^2 + m(0.32)^2$$ 5

$$Mh = m(0.32)$$ 5

$$2\pi\sqrt{\frac{I}{Mgh}} = 2\pi\sqrt{\frac{L}{g}}$$ 5

$$\frac{\frac{1}{3}m\ell^2 + m(0.32)^2}{m(0.32)} = 0.5$$

$$\ell = 0.416$$

Length of the rod = 0.83 m 5

(ii)

$$\frac{\frac{1}{3}m\ell^2 + mx^2}{mx} = 0.5$$

$$x^2 - 0.5x + 0.0576 = 0$$

$$x = 0.32 \quad \text{or} \quad x = 0.18$$

other distance = 18 cm 5

(iii)

$$T^2 = 4\pi^2 \frac{\left\{\frac{1}{3}m\ell^2 + mx^2\right\}}{mgx}$$

$$2T\frac{dT}{dx} = \frac{4\pi^2}{g}\left\{\frac{x(2x) - \left(\frac{1}{3}\ell^2 + x^2\right)(1)}{x^2}\right\}$$

$$= 0 \quad \text{when} \quad 2x^2 = \tfrac{1}{3}\ell^2 + x^2$$

$$\Rightarrow \quad x = \frac{\ell}{\sqrt{3}}$$

$$x = 0.24 \text{ m} \quad \text{or} \quad 24 \text{ cm}$$ 5