1999

6 (a) A particle moves with simple harmonic motion of period $\frac{\pi}{2}$. Initially it is 8 cm from the centre of the motion and moving away from the centre with a speed of $4\sqrt{2}$ cm/s. Find an equation for the position of the particle in time t seconds.

$$\text{Period} = \frac{2\pi}{\omega}$$

$$\frac{\pi}{2} = \frac{2\pi}{\omega}$$

$$\Rightarrow \quad \omega = 4 \qquad 5$$

$$v^2 = \omega^2(a^2 - x^2)$$

$$32 = 16\,(a^2 - 64)$$

$$\Rightarrow \quad a = \sqrt{66} \qquad 5$$

$x = 8,\ t = 0 \quad \Rightarrow \quad x = a\sin(\omega t + \varepsilon)$

$$8 = \sqrt{66}\sin\varepsilon$$

$$\Rightarrow \quad \varepsilon = 1.4 \text{ radians} \qquad 5$$

Equation $\quad x = \sqrt{66}\sin(4t + 1.4) \qquad 5$

Alternative method for finding the amplitude:

$x = a\sin(\omega t + \varepsilon) \quad \Rightarrow \quad 8 = a\sin\varepsilon$

$v = a\omega\cos(\omega t + \varepsilon) \quad \Rightarrow \quad \sqrt{2} = a\cos\varepsilon$

$$a^2\sin^2\varepsilon + a^2\cos^2\varepsilon = 64 + 2$$

$$a^2 = 66$$

$$\Rightarrow \quad a = \sqrt{66}$$