1998

8 (c) A uniform rod [ab], of mass m and length 2ℓ, is free to rotate in a vertical plane about a fixed horizontal axis at a, with a particle of mass $3m$ attached to the rod at b. The system is released from rest with the rod vertical and the end b above a.

(i) Show that the angular velocity of the rod when next it is vertical is $\sqrt{\frac{21g}{10\ell}}$.

(ii) If at this point the mass falls off, find the height to which the end b subsequently rises.

$I = \frac{4}{3}m\ell^2 + 3m(2\ell)^2$ 5

$= \frac{40m\ell^2}{3}$ 5

Gain in Kinetic Energy = Loss in Potential Energy

$\frac{1}{2}I\omega^2 = mg(2\ell) + (3m)g(4\ell)$

$\frac{20m\ell^2\omega^2}{3} = 14mg\ell$

$\Rightarrow \quad \omega = \sqrt{\frac{21g}{10\ell}}$ 5

(i)

Loss in Kinetic Energy = Gain in Potential Energy

$\frac{1}{2}\left(\frac{4}{3}m\ell^2\right)\left(\frac{21g}{10\ell}\right) = mgh$

$\Rightarrow \quad h = \frac{7\ell}{5}$

End b will rise a distance $2h = \frac{14\ell}{5}$ 5

20