1998

8 (a) Prove that the moment of inertia of a uniform rod [ab] of mass m and length 2ℓ about an axis through a, perpendicular to the rod, is $\frac{4}{3}m\ell^2$.

Let m_1 = mass per unit length

Mass of rod $m = 2m_1\ell$

Consider an element of the rod of width Δx, a distance x from the axis.

Mass of the element $= m_1\Delta x$ — 5

Moment of inertia $= \int_0^{2\ell} m_1 x^2 dx$ — 5

$= \frac{m_1}{3}\left[x^3\right]_0^{2\ell}$ — 5

$= \frac{8m_1\ell^3}{3}$

$= \frac{4m\ell^2}{3}$ — 5

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8 (b) A lamina is rotating with angular velocity ω about an axis perpendicular to its plane. If the moment of inertia of the lamina about the axis is I, prove that the kinetic energy is $\frac{1}{2}I\omega^2$.

Consider a particle of the body of mass m, a distance r from the axis.

Kinetic Energy of particle $= \frac{1}{2}mv^2$

$= \frac{1}{2}mr^2\omega^2$ — 5

Kinetic Energy of the lamina $= \sum \frac{1}{2}mr^2\omega^2$

$= \frac{1}{2}\omega^2 \sum mr^2$

$= \frac{1}{2}I\omega^2$ — 5

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