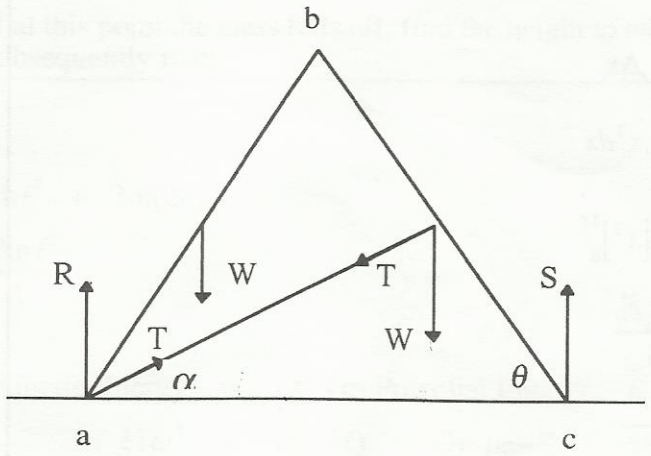
1998

7 Two equal uniform rods [ab] and [bc], each of weight W, are freely jointed at b. An inextensible string connects a to the midpoint of [bc]. When the string is taut the angle bca is θ. The rods are placed in a vertical plane with a and c on a smooth horizontal surface.

Prove that the tension in the string is $\frac{W}{4}\sqrt{1+9\cot^2\theta}$.



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Resolve vertically $R + S = 2W$ 5

Moments about a for system $W(1) + W(3) = S(4)$ 5

$\Rightarrow S = W$ and $R = W$ 5

Moments about b for ba

$T\cos\alpha.2\ell\sin\theta + W.\ell\cos\theta = T\sin\alpha.2\ell\cos\theta + R.2\ell\cos\theta$ 5,5

$2T\cos\alpha.\tan\theta + W = 2T\sin\alpha + 2W \quad (\because R = W)$

$\Rightarrow T\cos\alpha.\tan\theta = \frac{W}{2} + T\sin\alpha$ ..... eq (1)

Moments about b for bc

$T\cos\alpha.\ell\sin\theta + T\sin\alpha.\ell\cos\theta + W.\ell\cos\theta = S.2\ell\cos\theta$ 5,5

$T\cos\alpha.\tan\theta + T\sin\alpha + W = 2W \quad (\because S = W)$

$\Rightarrow T\cos\alpha.\tan\theta = W - T\sin\alpha$ ..... eq (2)

Solve equations (1) and (2)

$T\sin\alpha = \frac{W}{4}$ and $T\cos\alpha = \frac{3W}{4\tan\theta}$

$\Rightarrow T^2\sin^2\alpha + T^2\cos^2\alpha = \frac{W^2}{16} + \frac{9W^2}{16\tan^2\theta}$

$\Rightarrow T = \frac{W}{4}\sqrt{1 + 9\cot^2\theta}$ 5

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