6 (b) An elastic string of natural length one metre is extended 20 cm by a particle attached to its end and hanging freely. The particle is then pulled down a further distance of 40 cm and released.

(i) Show that the particle moves with simple harmonic motion when the string is taut

(ii) Find the height above the equilibrium position to which the particle will rise.

(i) Equilibrium position

$$T_0 = mg$$
$$k(0.2) = mg$$
$$\Rightarrow \quad k = 5mg \quad \text{or} \quad 49m$$

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Displaced position

Force in direction of x increasing $= mg - k(0.2 + x)$

$$= mg - mg - 5mgx$$
$$= -5mgx$$

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$$\text{acceleration} = -5gx$$
$$= -49x$$

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$$\Rightarrow \quad \text{S.H.M. about } x = 0 \text{ with } \omega = 7$$

(ii) Find velocity of particle when string is 1 m long

$$v = \omega\sqrt{a^2 - x^2}$$
$$= 7\sqrt{(0.4)^2 - (0.2)^2}$$
$$= 7\sqrt{0.12} \quad \text{or} \quad 2.43$$

Find distance

$$v^2 = u^2 + 2as$$
$$0 = 49(0.12) + 2(-9.8)s$$

$$\Rightarrow s = 0.3 \quad \Rightarrow \quad 0.5 \text{ m above the equilibrium position}$$

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