1998

5 (a) Two smooth spheres A and B have masses $m_1$ and $m_2$, respectively. They are moving towards each other along the same horizontal line each with speed $2u$. After collision both spheres reverse their original directions of motion and A now travels with speed $u$.

(i) Show that $3m_1 > 2m_2$.

(ii) Find an expression for e, the coefficient of restitution, and hence or otherwise show that $3m_1 \leq 5m_2$.

| | mass | velocity before | velocity after | | |
|---|---|---|---|---|---|
| A | $m_1$ | $2u$ | $-u$ | | |
| B | $m_2$ | $-2u$ | $v$ | | |

P.C.M. $m_1(2u) + m_2(-2u) = m_1(-u) + m_2(v)$ eq(1) 10

N.E.L. $v - (-u) = -e(-2u - 2u)$ eq(2) 10

(i) From eq(1) $v = \dfrac{3m_1u - 2m_2u}{m_2}$

$v > 0 \Rightarrow 3m_1 > 2m_2$ 5

(ii) From eq(2) $e = \dfrac{v + u}{4u}$

$e \leq 1 \Rightarrow v + u \leq 4u$

$\Rightarrow v \leq 3u$

$\Rightarrow \dfrac{3m_1u - 2m_2u}{m_2} \leq 3u$

$\Rightarrow 3m_1 \leq 5m_2$ 5

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