1998

3 (b) A particle is projected down a slope which is inclined at 45° to the horizontal. The particle is projected from a point on the slope and has an initial velocity of $7\sqrt{2}$ m/s at an angle α to the inclined plane.
Find the value of α if

(i) the particle first hits the slope after 2 seconds

(ii) the landing angle with the slope is $\tan^{-1}\left(\frac{1}{3}\right)$

$$\vec{r} = (7\sqrt{2}\cos\alpha.t + \tfrac{1}{2}g\sin 45.t^2)\,\vec{i} + (7\sqrt{2}\sin\alpha.t - \tfrac{1}{2}g\cos 45.t^2)\,\vec{j}$$

$$\vec{v} = (7\sqrt{2}\cos\alpha + g\sin 45.t)\,\vec{i} + (7\sqrt{2}\sin\alpha - g\cos 45.t)\,\vec{j}$$

Particle hits plane $\Rightarrow y_f = 0 \Rightarrow t = \dfrac{14\sqrt{2}\sin\alpha}{g\cos 45}$ 5

$$\Rightarrow 2 = \frac{28\sin\alpha}{g} \quad 5$$

$$\sin\alpha = \frac{g}{14} \quad \text{or} \quad \alpha = 44.43° \quad 5$$

(ii) $\tan(\text{landing angle}) = \dfrac{-V_j}{V_i}$ 5

$$\frac{1}{3} = \frac{\frac{g}{\sqrt{2}}t - 7\sqrt{2}\sin\alpha}{7\sqrt{2}\cos\alpha + \frac{g}{\sqrt{2}}t}$$

$$7\sqrt{2}\cos\alpha + \frac{g}{\sqrt{2}}\frac{28\sin\alpha}{g} = \frac{3g}{\sqrt{2}}\frac{28\sin\alpha}{g} - 21\sqrt{2}\sin\alpha$$

$$\tan\alpha = 1 \quad \text{or} \quad \alpha = 45° \quad 5$$

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cancel g's, Multiply top line by $\sqrt{2}$

$14\cos\alpha + 28\sin\alpha = 84\sin\alpha - 42\sin\alpha$

$\Rightarrow \cos\alpha = \sin\alpha$

$\Rightarrow \tan\alpha = 1$

$\Rightarrow \alpha = 45°$