1998

10 (b) A particle moves in a straight line. The initial speed is u and the retardation is kv^3, where v is the speed at the time t. If s is the distance travelled in time t, prove

(i) $v = \dfrac{u}{1 + ksu}$

(ii) $t = \dfrac{ks^2}{2} + \dfrac{s}{u}$.

$$v\frac{dv}{ds} = -kv^3$$

$$\int \frac{dv}{-v^2} = k\int ds$$ 5

$$\frac{1}{v} = ks + C$$ 5

$v = u$ when $s = 0 \Rightarrow \dfrac{1}{u} = C$ 5

$$\therefore \quad \frac{1}{v} = ks + \frac{1}{u}$$

$$= \frac{ksu + 1}{u}$$

$$\Rightarrow \quad v = \frac{u}{ksu + 1}$$ 5 — 20

$$\frac{ds}{dt} = \frac{u}{ksu + 1}$$

$$\int (ksu + 1)\, ds = u\int dt$$ 5

$$\tfrac{1}{2}ks^2u + s = ut + A$$

$t = 0$ when $s = 0 \quad \Rightarrow \quad 0 = A$

$$\therefore \quad \tfrac{1}{2}ks^2u + s = ut$$

$$\Rightarrow \quad t = \frac{ks^2}{2} + \frac{s}{u}$$ 5 — 10