1998

10 (a) If $t\frac{dv}{dt} = v - vt$ and $v = 3$ when $t = 5$,

find the value of v when $t = 6$.

$$\int \frac{dv}{v} = \int \left(\frac{1 - t}{t}\right) dt \qquad 5$$

$$\ln v = \ln t - t + C \qquad 5$$

$v = 3$ when $t = 5$ $\Rightarrow$ $\ln 3 = \ln 5 - 5 + C$ 5

$\Rightarrow$ $C = 5 + \ln \frac{3}{5}$ or 4.489

$\therefore$ $\ln v = \ln t - t + 5 + \ln \frac{3}{5}$

Find v when $t = 6$ $\Rightarrow$ $\ln v = \ln 6 - 6 + 5 + \ln \frac{3}{5}$

$\Rightarrow$ $\ln v = \ln \frac{18}{5} - 1$ or 0.281

$\Rightarrow$ $v = 1.32$ or $\frac{18}{5e}$ or $e^{0.281}$ 5

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