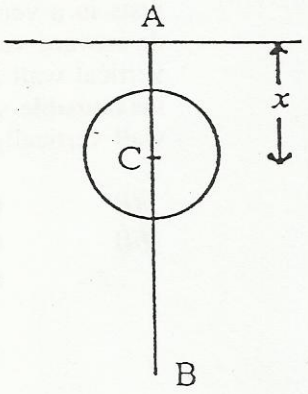
1998 1997

8. *(a)* Prove that the moment of inertia of a uniform circular disc, of mass m, and radius r, about an axis through its centre perpendicular to its plane is $\frac{1}{2}mr^2$.

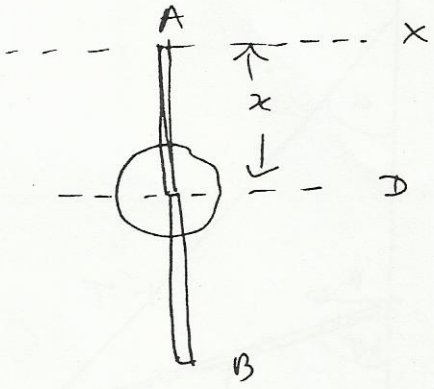
(b) A thin uniform rod AB of mass m, and length $2a$ can turn freely in a vertical plane, about a fixed horizontal axis through A. A uniform circular disc of mass $24m$ and radius $a/3$ has its centre C clamped to the rod so that the length AC = x and the plane of the disc passes through the axis of rotation.



(i) Show that the moment of inertia of the system about the axis is $2m(a^2 + 12x^2)$.

(ii) The system makes small oscillations. Find the period and show that the period is a minimum when $x = a/4$.

(b) Rod: "$\frac{4}{3}ml^2$" $= \frac{4}{3}ma^2$



Disc: $I_D =$ "$\frac{1}{4}mr^2$"

$= \frac{1}{4}(24m)\left(\frac{a}{3}\right)^2$

$= 6m\left(\frac{a^2}{9}\right)$

$= \frac{2}{3}ma^2$

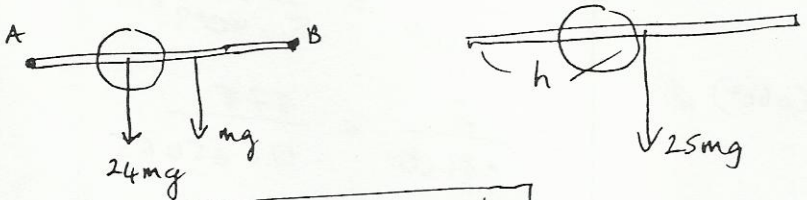
$I_x = \frac{2}{3}ma^2 + 24m(x^2)$

TOTAL $= \frac{4}{3}ma^2 + \frac{2}{3}ma^2 + 24mx^2$

$= 2ma^2 + 24mx^2$

$= 2m(a^2 + 12x^2)$.

(ii)



$24mg(x) + mga = 25mgh$