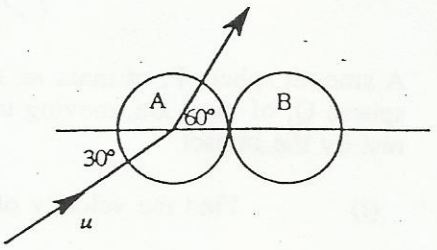
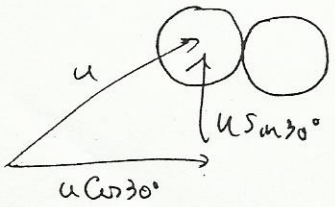
(b) 1997 5(b)

(b) A smooth sphere A, of mass m, moving with speed u collides with a smooth sphere B, of mass m, which is at rest. The direction of motion of A before impact makes an angle of 30° with the line of centres. If the coefficient of restitution between the spheres is e, find



(i) the velocity of each sphere after impact

(ii) the value of e if after impact the direction of A makes an angle of 60° with the line of centres.



$\frac{\sqrt{3}}{2}u\vec{i} + \frac{1}{2}u\vec{j}$ (m) $\quad p\vec{i} + \frac{1}{2}u\vec{j}$

$0\vec{i} + 0\vec{j}$ (m) $\quad q\vec{i} + 0\vec{j}$

① $m\left(\frac{\sqrt{3}}{2}u\right) + m(0) = mp + mq$

$p + q = \frac{\sqrt{3}u}{2}$

② $\frac{p-q}{\frac{\sqrt{3}u}{2} - 0} = -e \Rightarrow p - q = -\frac{\sqrt{3}eu}{2}$

$p + q = \frac{\sqrt{3}u}{2}$

$p - q = -\frac{\sqrt{3}eu}{2}$

$2p = \frac{\sqrt{3}u}{2}(1-e)$

$p = \frac{\sqrt{3}u}{4}(1-e)$

Re-solve!

$p + q = \frac{\sqrt{3}u}{2}$

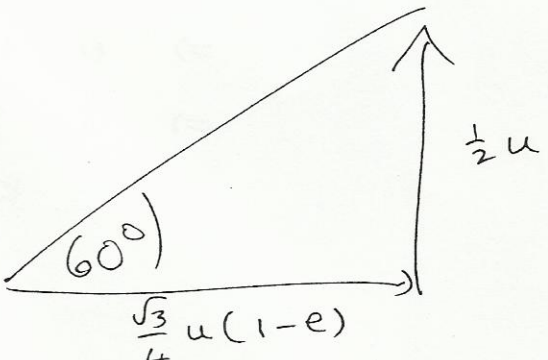
$-p + q = \frac{\sqrt{3}eu}{2}$

$2q = \frac{\sqrt{3}}{2}u(1+e)$

$q = \frac{\sqrt{3}}{4}u(1+e)$

$\vec{v_1} = \frac{\sqrt{3}}{4}u(1-e)\vec{i} + \frac{1}{2}u\vec{j}$

$\vec{v_2} = \frac{\sqrt{3}}{4}u(1+e)\vec{i} + 0\vec{j}$



$\tan 60° = \frac{\frac{1}{2}u}{\frac{\sqrt{3}}{4}u(1-e)} = \sqrt{3}$

$\Rightarrow \frac{1}{2}u = \frac{3}{4}u(1-e)$

$\Rightarrow 2 = 3(1-e)$

$\Rightarrow \frac{2}{3} = 1-e$

$\Rightarrow e = \frac{1}{3}$