2

1997

2. To a man walking North at 4 m/s a constant wind appears to have a speed of 3 m/s. The man then changes direction and when he is walking West at 3 m/s the wind appears to have a speed of 4 m/s. There are two possible wind velocities.

If $\vec{i}$ and $\vec{j}$ are unit vectors in the East and North directions, respectively:

(i) show that $-3\vec{i} + 4\vec{j}$ is one possible wind velocity,

(ii) find the second possible wind velocity.

$U_m = 4\vec{j}$

$U_w = x\vec{i} + y\vec{j}$

$\vec{U}_{wm} = \vec{U}_w - \vec{U}_m = x\vec{i} + (y-4)\vec{j}$

The relative speed $= \sqrt{x^2+(y-4)^2} = 3$

$\Rightarrow x^2 + (y-4)^2 = 9 \Rightarrow x^2+y^2-8y+16=9$

$\Rightarrow x^2+y^2-8y+7=0$ --- A

$\vec{U}_m = -3\vec{i}$

$\vec{U}_w = x\vec{i} + y\vec{j}$

$\vec{U}_{wm} = \vec{U}_w - \vec{U}_m = (x+3)\vec{i} + y\vec{j}$

appart speed $= \sqrt{(x+3)^2+y^2} = 4$

$\Rightarrow (x+3)^2 + y^2 = 16 \Rightarrow x^2+6x+9+y^2 = 16$

$\Rightarrow x^2+y^2+6x-7=0$ --- B

Add:
$\cancel{x^2}+\cancel{y^2}-8y+7=0$
$-\cancel{x^2}-\cancel{y^2}-6x+7=0$

$-8y-6x+14=0$

$\therefore 6x = \frac{14-8y}{6}$

$x = \frac{7-4y}{3}$

$x^2+y^2-8y+7=0$

$\left(\frac{7-4y}{3}\right)^2+y^2-8y+7=0$

$\frac{49-56y+16y^2}{9}+y^2-8y+7=0$

$49-56y+16y^2+9y^2-72y+63=0$

$25y^2-128y+112=0$

$(y-4)(25y-28)=0$

$\therefore y=4 \quad y=1.12$

[we know $y=4$ is one answer

NS { If $y=4$, $x=\frac{7-16}{3}=-3$: $-3\vec{i}+4\vec{j}$

If $y=1.12$, $x=0.84$: $0.84\vec{i}+1.12\vec{j}$