1997

Q1

1. (a) A particle, moving in a straight line, accelerates uniformly from rest to a speed v m/s. It continues at this constant speed for a time and then decelerates uniformly to rest, the magnitude of the deceleration being twice that of the acceleration. The distance travelled while accelerating is 6 m. The total distance travelled is 30 m and the total time taken is 6 s.

(i) Draw a speed-time graph and hence, or otherwise, find the value of v.

(ii) Calculate the distance travelled at v m/s.

(b) Two points p and q are a distance d apart. A particle starts from p and moves towards q with initial velocity $2u$ and uniform acceleration f. A second particle starts at the same time from q and moves towards p with initial velocity $3u$ and uniform deceleration f. Prove that

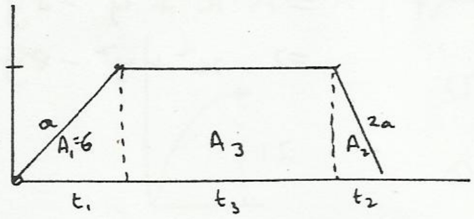
(i) the particles collide after $\dfrac{d}{5u}$ seconds,

(ii) if the particles collide before the second particle comes to instantaneous rest, then

$$fd < 15u^2,$$

(iii) if $fd = 30u^2$ then the second particle has returned to q before the collision.

(a)



$t_1 : t_2 = 2:1$

$\therefore$ Area$_1$: Area 2 = 2 : 1

$\therefore A_1 = 6 \therefore A_2 = 3$

$\therefore A_3 = 30 - 3 - 6 = 21$ = Distance travelled at v

$\frac{1}{2} t_1 v = 6 \quad \therefore t_1 = \frac{12}{v}$

$\frac{1}{2} t_2 v = 3 \quad \therefore t_2 = \frac{6}{v}$

$t_3 v = 21 \quad \therefore t_3 = \frac{21}{v}$

Total time = 6

$\therefore \frac{12}{v} + \frac{6}{v} + \frac{21}{v} = 6$

$\therefore 12 + 6 + 21 = 6v$

$\therefore 39 = 6v$

$\therefore 13 = 2v$

$\therefore v = \frac{13}{2}$

(ii) Distance travelled at v m/s is 21 m.

(b) (i)

$s_1 = 2ut + \frac{1}{2}ft^2$

$s_2 = 3ut - \frac{1}{2}ft^2$

They collide when

$s_1 + s_2 = d$

$\therefore 5ut = d$

$\therefore t_1 = \frac{d}{5u}$

(ii) Let t_2 = time when 2nd particle comes to rest:

$\therefore 3u - ft_2 = 0$

$\therefore t_2 = \frac{3u}{f}$

Collision before Rest

$\therefore t_1 < t_2$

$\frac{d}{5u} < \frac{3u}{f}$

$\therefore fd < 15u^2$

(iii) 2nd particle gets back to q when $s_2 = 0$

$\therefore 3ut - \frac{1}{2}ft^2 = 0$

$\therefore 3u - \frac{1}{2}ft = 0$

$\therefore t = \frac{6u}{f}$

$\frac{6u}{f} = \frac{d}{5u}$ (2nd part back at same time)

$\therefore fd = 30u^2$