1997 10(b)

(b) A particle moves in a straight line and undergoes a retardation of $0.04v^3$ m/s^2, where v is its speed.

(i) If the initial speed of the particle is 25 m/s, find its speed when it has travelled a distance of 49 m.

(ii) Find the time for the speed to reduce from 25 m/s to 15 m/s.

(i) $a = -0.04v^3$

$v\frac{dv}{ds} = -0.04v^3$

$\int_{25}^{v}\frac{dv}{v^2} = \int_0^{49} -0.04\,ds$

$\frac{v^{-1}}{-1}\Big|_{25}^{v} = -0.04s\Big|_0^{49}$

$-\frac{1}{v}\Big|_{25}^{v} = -0.04s\Big|_0^{49}$

$-\frac{1}{v} + \frac{1}{25} = -1.96$

$-\frac{1}{v} = -1.96 - 0.04$

$-\frac{1}{v} = -2$

$\frac{1}{v} = 2$

$v = \frac{1}{2}$

$\frac{dv}{dt} = -0.04v^3$

$\int v^{-3}\,dv = \int -0.04\,dt$

$\frac{v^{-2}}{-2}\Big|_{25}^{15} = -0.04t\Big|_0^{t}$

$-\frac{1}{2v^2}\Big|_{25}^{15} = -0.04t - 0$

$-\frac{1}{450} + \frac{1}{1250} = -0.04t$

$\frac{10}{4500} - \frac{10}{12500} = +4t$

$\frac{32}{225} = 4t$

$t = \frac{8}{225} = 0.0\dot{3}5\,s.$