

1997

10. (a) If

$$\frac{dy}{dx} + x + xy = 0$$

and $y = 2$ when $x = 0$, find, correct to two places of decimals, the value of y when $x = 1$.

$$\frac{dy}{dx} = -x(1+y)$$

$$\int \frac{dy}{1+y} = \int -x \, dx$$

$$\log_e(1+y) = -\frac{x^2}{2} + c$$

$$\left[\log_e 3 = c \right]$$

$$\Rightarrow \log_e(1+y) = -\frac{x^2}{2} + \log_e 3$$

$$\text{When } x=1 \Rightarrow \log_e(1+y) = -\frac{1}{2} + \log_e 3$$

$$\Rightarrow \log_e\left(\frac{1+y}{3}\right) = -\frac{1}{2}$$

$$\Rightarrow \frac{1+y}{3} = e^{-\frac{1}{2}}$$

$$\Rightarrow y = 3e^{-\frac{1}{2}} - 1$$

$$\Rightarrow y = 3(0.60653) - 1 = 0.82$$