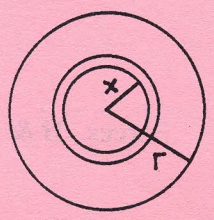
1996

8 (a)



Let m = mass per unit area

mass of element = m $(2\pi x.dx)$

Moment of Inertia of element = $(2\pi m\, x.dx)\, x^2$ 5

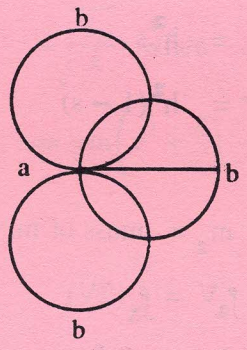
$$I = 2\pi m \int_0^r x^3\, dx \qquad 5$$

$$= 2\pi m \left[\frac{x^4}{4}\right]_0^r \qquad 5$$

$$= \pi m \frac{r^4}{2}$$

$$= \frac{M r^2}{2} \quad \text{where } M = m\pi r^2 \qquad 5 \quad 20$$

(b)



(i) $I = 0.5\, m r^2 + m r^2 = \frac{3}{2} m r^2$ 5

Gain in K.E. = Loss in P.E.

$$0.5\, I \omega^2 - 0.5\, I \frac{p^2}{4r^2} = m g r \qquad 5$$

$$\frac{3}{4} m r^2 \omega^2 - \frac{3}{4} m r^2 \frac{p^2}{4r^2} = m g r \qquad 5$$

$$\omega = \sqrt{\frac{16 g r + 3 p}{12 r}} \qquad 5 \quad 20$$

(ii) Loss in K.E. = Gain in P.E.

$$\frac{1}{2} \frac{3}{2} m r^2 \frac{p^2}{4r^2} = mgr \qquad 5$$

$$p = \sqrt{\frac{16gr}{3}} \text{ or } 4\sqrt{\frac{gr}{3}} \qquad 5 \quad 10$$