1996

10 (a)

$$\int \frac{dy}{y} = 4\int \cos x\,dx \qquad 5$$

$$\ln y = 4\sin x + c \qquad 5$$

$$y = e^2 \text{ when } x = \frac{\pi}{6} \Rightarrow 2 = 4(\tfrac{1}{2}) + c$$

$$\Rightarrow c = 0 \qquad 5$$

$$\therefore \ln y = 4\sin x$$

$$\Rightarrow y = e^{4\sin x} \qquad 5 \qquad 20$$

(b) (i) particle moving upwards

$$mv\frac{dv}{dx} = -mkv^2 - mg$$

$$\int\left(\frac{v\,dv}{v^2 + \frac{g}{k}}\right) = -k\int dx$$

$$\tfrac{1}{2}\ln\left(v^2 + \tfrac{g}{k}\right) = -kx + C \qquad 5$$

$$v = \sqrt{\frac{2g}{k}} \text{ when } x = 0 \Rightarrow \tfrac{1}{2}\ln\left(\frac{3g}{k}\right) = 0 + C$$

$$\Rightarrow \tfrac{1}{2}\ln\left(v^2 + \frac{g}{k}\right) = \tfrac{1}{2}\ln\left(\frac{3g}{k}\right) - kx \qquad 5$$

Find x when $v = 0$

$$\tfrac{1}{2}\ln\left(\frac{g}{k}\right) = \tfrac{1}{2}\ln\left(\frac{3g}{k}\right) - kx$$

$$kx = \tfrac{1}{2}\ln 3$$

$$x = \frac{\ln 3}{2k} \qquad 5 \qquad 15$$