1995

10(a) $\int y\,dy = \int \frac{4}{1+x^2}\,dx$ 5

$0.5y^2 = 4\tan^{-1}x + A$ 5

x=0, y=1 $\Rightarrow$ $0.5 = 4(0) + A$ 5

$\therefore\ 0.5y^2 = 4\tan^{-1}x + 0.5$ 5 20

or $y^2 = 8\tan^{-1}x + 1$

1995 (b) (i) $m\frac{dv}{dt} = mg - mkv$

$\int \frac{dv}{g-kv} = \int dt$ 5

$-\frac{1}{k}\ln(g-kv) = t + A$ 5

$v = 0, t = 0 \Rightarrow -\frac{1}{k}\ln(g) = A$ 5

$\therefore\ -\frac{1}{k}\ln(g-kv) + \frac{1}{k}\ln(g) = t$ 5

$\Rightarrow\ t = \frac{1}{k}\ln\left(\frac{g}{g-kv}\right)$ eq1

when $v = \frac{g}{2k}$ $t = \frac{1}{k}\ln 2$ 5 25

(ii) From equation 1

$\frac{g}{g-kv} = e^{kt}$

$g - kv = g e^{-kt}$

$v = \frac{g}{k}(1 - e^{-kt})$

$\rightarrow \frac{g}{k}$ as $t \rightarrow \infty$ 5 5