1994

9(a) (i) mass of sea-water displaced = 200 tonnes

volume of water displaced = 200000÷1030 = 194.17 m^3

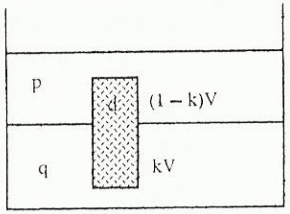
(ii) This volume of fresh water has a mass = 194.17 tonnes

⇒ Submarine must reduce its mass to 194.17 tonnes

by pumping out 5.83 tonnes of water from ballast tanks

(b) Let k be the fraction of the volume of the solid immersed in the lower liquid (of density q).

⇒ (1 − k) is the fraction in the upper liquid (of density p)



$$B_p + B_q = W$$

$$\frac{(1-k)Vdg\,\frac{p}{1000}}{d/1000} + \frac{kVdg\,\frac{q}{1000}}{d/1000} = Vdg$$

⇒ $(1-k)p + kq = d$

$$k = \frac{d-p}{q-p}$$

1994

10(a)

$$\int \frac{dy}{y} = \int \left(\frac{1-x}{1+x}\right) dx$$ ← difficult ,

$$= \int \left(-1 + \frac{2}{1+x}\right) dx$$

$\ln y = -x + 2\ln(1+x) + C$

$\ln 1 = -0 + 2\ln(1+0) + C$

$C = 0$

⇒ $\ln y = 2\ln(1+x) - x$

$y = e^{2\ln(1+x) - x}$ or $y = (1+x)^2 e^{-x}$

(b) (i) Power = Tractive force x velocity

Poorly done correctly

$75000 = T\,v$

Force = mass x acceleration

$T - 1500 = 1000\,f$

$\frac{75000}{v} - 1500 = 1000\,f$

⇒ $f = \frac{75}{v} - 1.5 = \frac{150 - 3v}{2v}$

(ii) $\frac{dv}{dt} = \frac{3(50-v)}{2v}$

$$\int_0^{25} \frac{v\,dv}{50-v} = 1.5\int_0^t dt$$

$$\int_0^{25} \left(-1 + \frac{50}{50-v}\right) dv = 1.5\int_0^t dt$$

$$\Big[-v - 50\ln(50-v)\Big]_0^{25} = 1.5\,t$$

⇒ $t = 6.44$ seconds

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