

10 (a)

$$(x^2 + 2) \frac{dy}{dx} = x(y + 1)$$

$$\int \frac{dy}{y+1} = \int \frac{x dx}{x^2 + 2}$$

$$\ln(y+1) = 0.5 \ln(x^2 + 2) + C$$

$$y=2, x=1 \Rightarrow \ln 3 = 0.5 \ln 3 + C$$

$$C = 0.5 \ln 3$$

$$\ln(y+1) = 0.5 \ln(x^2 + 2) + 0.5 \ln 3$$

$$y+1 = \sqrt{3x^2 + 6}$$

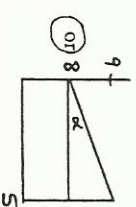
$$x=2 \Rightarrow y+1 = \sqrt{18}$$

$$y = 3\sqrt{2} - 1 \quad \text{or} \quad 3.24$$

(b) (i) retardation = $8 + kx$

$$9 = 8 + k(5)$$

$$k = 0.2$$



$$\tan \alpha = 0.2$$

$$y = 0.2x + 8$$

$$v \frac{dv}{dx} = -\left(8 + \frac{x}{5}\right)$$

$$\int_{20}^0 v dv = -\int_0^{x_1} \left(8 + \frac{x}{5}\right) dx$$

$$\left[0.5v^2\right]_{20}^0 = -\left[8x + \frac{x^2}{10}\right]_{x_1}^{x_1}$$

$$-200 = -8x_1 - 0.1x_1^2$$

$$x_1^2 + 80x_1 - 2000 = 0$$

$$(x_1 - 20)(x_1 + 100) = 0$$

$$x_1 = 20 \text{ metres}$$

B = W

$$\frac{17W(1)}{20s} = W$$

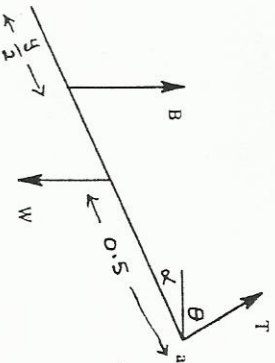
$$s = \frac{17}{20}$$

Let x = depth of layer of oil; A = cross-sectional area

$$B_{\text{water}} + B_{\text{oil}} = W$$

$$1000A(20-x)g + 800Axg = 850A(20)g \quad \text{or} \quad \frac{(20-x)W(1)}{20} + \frac{xW(0.8)}{17/20} = W$$

$$x = 15 \text{ cm.}$$



(b) (i)

$$\text{horiz: } T \cos \theta = 0$$

$$\Rightarrow \cos \theta = 0$$

$$\Rightarrow \theta = 90^\circ$$

(ii)

$$B = \frac{yW(1)}{0.64}$$

moments about a:

$$W(0.5) \cos \alpha = B(1 - 0.5y) \cos \alpha$$

$$0.32 = y - 0.5y^2$$

$$y^2 - 2y + 0.64 = 0$$

$$(y - 0.4)(y - 1.6) = 0$$

$$y = 0.4 \text{ m.}$$

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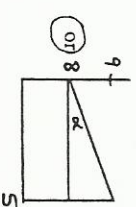
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