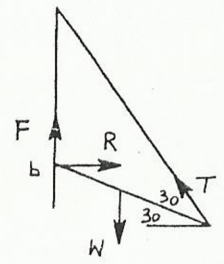
H93 1993 no. 7



moments about b:

$$T\sin 30 . 2\ell = W \sin 60 . \ell$$

$$T = \frac{W\sqrt{3}}{2}$$

horiz: $R = T\cos 60$ or $\frac{W\sqrt{3}}{4}$

vert: $\mu R + T\sin 60 = W$ or $F + T\sin 60 = W$

$$\mu . \frac{W\sqrt{3}}{4} + \frac{W\sqrt{3}}{2}.\frac{\sqrt{3}}{2} = W$$

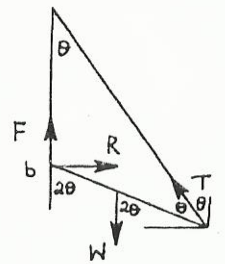
$$\mu = \frac{1}{\sqrt{3}}$$

$F = W/4$

Equilibrium if $F \le \mu R$

$$\mu \ge \frac{1}{\sqrt{3}}$$

(ii)



moments about b:

$$T\sin\theta . 2\ell = W \sin 2\theta . \ell$$

$$T = W\cos\theta$$

horiz: $R = T\sin\theta$ or $W\sin\theta\cos\theta$

vert: $F + T\cos\theta = W$

$$F = W - W\cos\theta.\cos\theta = W\sin^2\theta$$

Rod slips if $F > \mu R$

$W\sin^2\theta > \mu W\sin\theta\cos\theta$

$\tan\theta > \mu$ or $\theta > 30°$

$\frac{F}{R} = \frac{W\sin^2\theta}{W\sin\theta\cos\theta} = \tan\theta$

$> \tan 30°$ (or μ)

8 (a) Let m = mass per unit length

mass of element = mdx

moment of inertia of element = $(mdx)x^2$

$$I = \int_{-\ell}^{\ell} mx^2\,dx = m\left[\frac{x^3}{3}\right]_{-\ell}^{\ell} = \frac{2m}{3}\ell^3 = \frac{1}{3}M\ell^2 \quad \text{where } M = m.2\ell$$

(b) (i) $I = \frac{1}{3}m(0.6)^2 + m(0.2)^2$ or $0.16m$

Gain in K.E. = Loss in P.E.

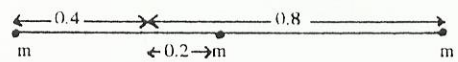
$$\frac{1}{2}I\omega^2 = mgh$$

$$0.08\,m\,\omega^2 = mg(0.4)$$

$$\omega = 7 \text{ rad/s}$$

$v = r\omega = 0.8(7) = 5.6$ m/s

(ii)



$I = 0.16m + m(0.8)^2 + m(0.4)^2$ or $0.96m$

$Mgh = mg(0.2) + mg(0.8) - mg(0.4)$ or $0.6mg$

$$T = 2\pi\sqrt{\frac{I}{Mgh}} = 2\pi\sqrt{\frac{0.96m}{0.6mg}} = 2.54 \text{ seconds}$$