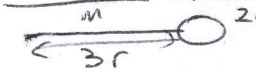
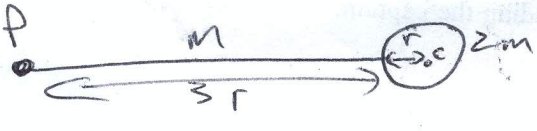
MLC Q8 1994 RBM



(i) $T = 2\pi\sqrt{\dfrac{I}{\text{Mass} \times g \times |\text{dist axis to c of g}|}}$

① Mass $= m + 2m = 3m$

② I_P



$I_{P\text{-}O} = I_P + I_{P\,\odot C}$

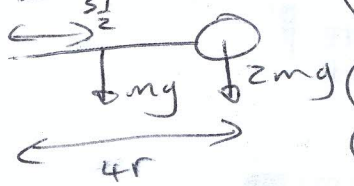
$= \frac{4}{3}m\left(\frac{3r}{2}\right)^2 + \left[I_C + (2m)|PC|^2\right]$

$= 3mr^2 + \left[\frac{1}{2}(2m)r^2 + 2m(4r)^2\right]$

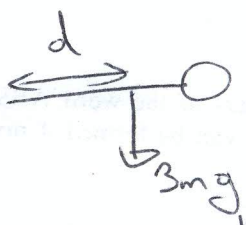
$= 3mr^2 + mr^2 + 32mr^2$

$= 36mr^2$

(3) dist:



$\mathcal{E}_{man} = mg\frac{3r}{2} + 2mg(4r)$



man $\Sigma = 3mgd$

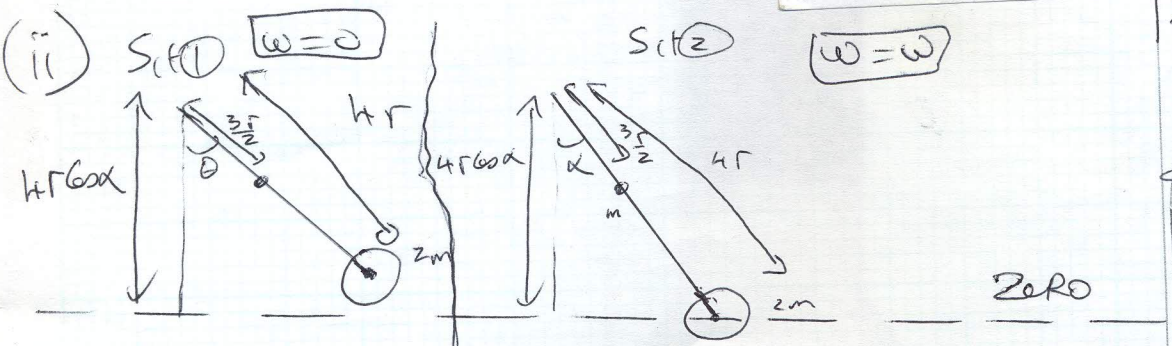
$\therefore \frac{19r}{2}mg = 3mgd$

$\frac{19r}{6} = d$

$\therefore T = 2\pi\sqrt{\dfrac{36mr^2}{3m \cdot g \frac{19r}{6}}}$

$T = 2\pi\sqrt{\dfrac{72r}{19g}}$.

(ii)



$\mathcal{E}_1 = PE_{Rod} + PE_{Pism} + KE$

$= mg\left[4r\cos\alpha - \frac{3r}{2}\cos\theta\right] + 2mg\left[4r\cos\alpha - 4r\cos\theta\right] + 0$

$\mathcal{E}_1 = 12mgr\cos\alpha - \frac{19}{2}mgr\cos\theta$

$\mathcal{E}_2 = PE_{Rod} + PE_{Disk} + KE$

$= mg\left[4r\cos\alpha - \frac{3r}{2}\cos\alpha\right] + 2mg(0) + \frac{1}{2}I\omega^2$

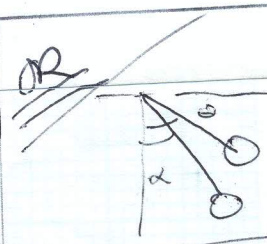
$\mathcal{E}_2 = \frac{5mgr}{2}\cos\alpha + \frac{1}{2}(36mr^2)\omega^2$

PCE $\Rightarrow \boxed{\mathcal{E}_1 = \mathcal{E}_2}$

$\Rightarrow 12mgr\cos\alpha - \frac{19}{2}mgr\cos\theta = \frac{5mgr}{2}\cos\alpha + 18mr^2\omega^2$

$\frac{19mgr}{2}\cos\alpha - \frac{19mgr}{2}\cos\theta = 18mr^2\omega^2$

$\frac{19g}{36r}(\cos\alpha - \cos\theta) = \omega^2$ qed.



Sum in KE = Loss in PE's.

$\frac{1}{2}I\omega^2 = \frac{3}{2}mgr(\cos\alpha - \cos\theta) + 2mg\,4r(\cos\alpha - \cos\theta)$

$\Rightarrow 18mr^2\omega^2 = \frac{19}{2}mg(\cos\alpha - \cos\theta)$

$\Rightarrow \omega^2 = \frac{19g}{36r}(\cos\alpha - \cos\theta)$.