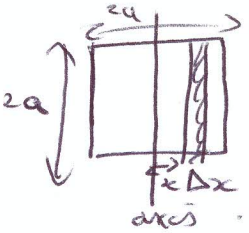
1990 [mqI] Q8



(a) Mass $\Delta m = \rho \Delta x \cdot 2a$

Mass $M = \rho(2a)(2a)$

$= 4a^2\rho$

$\Delta I = \Delta m \cdot x^2$

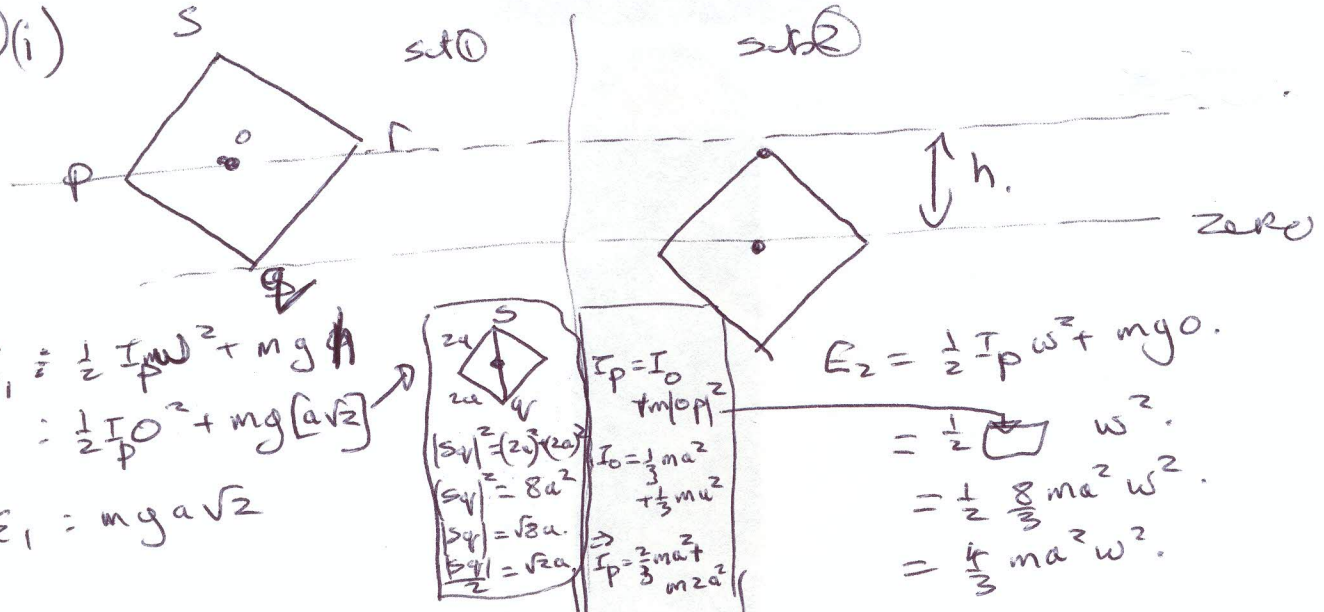
$\Delta I = (\rho \Delta x\, 2a)\cdot x^2$

$I = \int dI = \int_{-a}^{a} \rho 2a\, x^2 dx$

$= \rho 2a \int_{-a}^{a} x^2 dx$

$= \rho 2a \left.\frac{x^3}{3}\right|_{-a}^{a} = \frac{\rho 4a^3}{3} = \frac{1}{3} M a^2.$

(b)(i)



$E_1 = \frac{1}{2} I_p \omega^2 + mgh$

$= \frac{1}{2} I_p 0^2 + mg[a\sqrt{2}]$

$E_1 = mga\sqrt{2}$

$|SQ|^2 = (2a)^2 + (2a)^2$

$|SQ|^2 = 8a^2$

$|SQ| = \sqrt{8}a.$

$\frac{|SQ|}{2} = \sqrt{2}a.$

$I_p = I_0 + m|OP|^2$

$I_0 = \frac{1}{3}ma^2 + \frac{1}{3}ma^2$

$\Rightarrow I_p = \frac{2}{3}ma^2 + m2a^2$

$E_2 = \frac{1}{2} I_p \omega^2 + mgo.$

$= \frac{1}{2}\left[\frac{2}{3}ma^2 + m2a^2\right]\omega^2.$

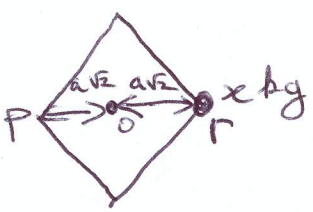
$= \frac{1}{2}\frac{8}{3}ma^2\omega^2.$

$= \frac{4}{3}ma^2\omega^2.$

PCE $\Rightarrow$ $E_1 = E_2 \Rightarrow mga\sqrt{2} = \frac{4}{3}ma^2\omega^2.$

$\Rightarrow \sqrt{\frac{3\sqrt{2}g}{4a}} = \omega$ (Weird!)

(ii) Attach extra x kg at r



$T = 2\pi\sqrt{\frac{I}{\text{mass} \times g \times h.}}$

System mass $= M + x$

$h = ?$

$\Rightarrow (M+x)h = M(a\sqrt{2}) + x(a\sqrt{2}) \Rightarrow h = a\sqrt{2}\frac{(M+2x)}{(M+x)}$

I of system $= \frac{8}{3}ma^2 + x(2\sqrt{2}a)^2 = \frac{8}{3}ma^2 + 8xa^2 = 8a^2\left(\frac{m}{3}+x\right)$

$\therefore$ Old $T = 2\pi\sqrt{\frac{8a}{3g}} = 2\pi\sqrt{\frac{8a^2\left(\frac{m}{3}+x\right)}{a\sqrt{2}(m+2x)g}}$ this is h. mass.

$\Rightarrow \frac{1}{3} = \frac{\frac{m}{3}+x}{\sqrt{2}(m+2x)}$

$\Rightarrow \frac{\sqrt{2}}{3}(m+2x) = \frac{m}{3}+x.$

$\Rightarrow \frac{\sqrt{2}}{3}m + \frac{2\sqrt{2}x}{3} = \frac{m}{3}+x.$

$\Rightarrow \sqrt{2}m + 2\sqrt{2}x = m + 3x$

$\Rightarrow \sqrt{2}m - m = (3-2\sqrt{2})x \Rightarrow x = \frac{(\sqrt{2}-1)}{(3-2\sqrt{2})}m$