APPENDIX ①

$$\frac{d^2x}{dt^2} = -\omega^2 x \text{ with } t=0, x=0$$
$$v = \frac{dx}{dt} = 0, x = A.$$

$$\Rightarrow v\frac{dv}{dx} = -\omega^2 x$$
$$\Rightarrow \int v\,dv = \int -\omega^2 x\,dx$$
$$\Rightarrow \frac{v^2}{2} = -\omega^2\frac{x^2}{2} + C$$
$$// x = A, v = 0 \Rightarrow$$
$$\Rightarrow \frac{\omega^2 A^2}{2} = C //$$
$$\therefore \frac{v^2}{2} = -\omega^2\frac{x^2}{2} + \frac{\omega^2 A^2}{2}$$
$$\Rightarrow \boxed{v^2 = \omega^2(A^2 - x^2)}$$
$$\Rightarrow v = \mp\omega\sqrt{A^2 - x^2}$$
$$\Rightarrow v = \omega\sqrt{A^2 - x^2}$$
$$\Rightarrow \frac{dx}{dt} = \omega\sqrt{A^2 - x^2}$$
$$\Rightarrow \int \frac{dx}{\sqrt{A^2 - x^2}} = \int \omega\,dt$$
$$\Rightarrow \sin^{-1}\frac{x}{A} = \omega t + C_1$$
$$// t = 0, x = 0 \Rightarrow 0 = 0 + C_1$$
$$0 = C_1 //$$
$$\Rightarrow \sin^{-1}\left(\frac{x}{A}\right) = \omega t$$
$$\Rightarrow \frac{x}{A} = \sin(\omega t)$$
$$\Rightarrow \boxed{x = A\sin\omega t}$$

APPENDIX ②

$$\frac{d^2x}{dt^2} = a \text{ with } t=0, x=0$$
$$v = u, t = 0$$

$$\Rightarrow \frac{dv}{dt} = a$$
$$\Rightarrow \int dv = \int a\,dt$$
$$\Rightarrow v = at + C$$
$$// v = u, t = 0 \Rightarrow u = 0 + C$$
$$u = C //$$
$$\Rightarrow \boxed{v = at + u}$$
$$\Rightarrow \frac{dx}{dt} = at + u$$
$$\Rightarrow \int dx = \int (at + u)\,dt$$
$$\Rightarrow x = a\frac{t^2}{2} + ut + C_1$$
$$// t = 0, x = 0 \Rightarrow 0 = C_1 //$$
$$\Rightarrow x = \frac{1}{2}at^2 + ut$$
$$\Rightarrow \boxed{x = ut + \frac{1}{2}at^2}$$