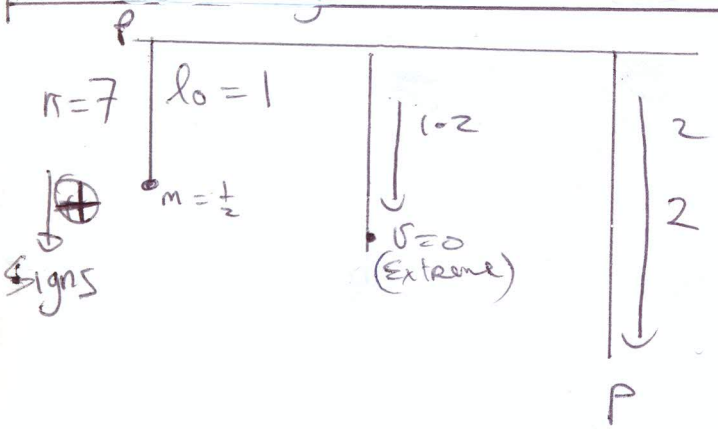
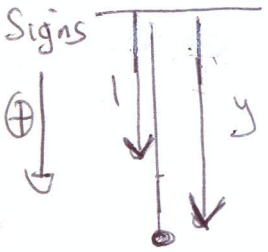
'Vertical' Examples

Eg1 A particle of mass ½ kg hangs vertically from a point p on the ceiling by means of a light elastic string of natural length 1 m and elastic constant 7 N/m. It is released from rest at a point 1.2 m below the ceiling. Show it will perform S.H.M. and calculate how long it takes to reach apoint 2 m below p.



Show SHM:

First find equilibrium y metres below ceiling.



Forces

Hooke $|S| = 7(y-1)$

At equilib. $\boxed{a=0}$

NII ⇒ Nett F = 0

$\Rightarrow -7(y-1) + \frac{1}{2}(9.8) = 0$

$\Rightarrow -7y + 7 + 4.9 = 0$

$\Rightarrow 7y = 11.9$

$\Rightarrow y = 1.7$

∴ Equil. position is 1.7 metres below ceiling.

Next Examine forces at typical position $1.7+x$ below ceiling

Forces Hooke $|S'| = 7(1.7+x-1)$

NII ⇒ Nett F = ma

$\Rightarrow -7(1.7+x-1) + \frac{1}{2}g = \frac{1}{2}a$

$\Rightarrow -7(.7+x) + 4.9 = \frac{1}{2}a$

$\Rightarrow -7x = \frac{1}{2}a$

$\Rightarrow -14x = a$

SHM about point 1.7 below ceiling with $\omega = \sqrt{14}$.

To find A: v=0 at 1.2 below ceiling

$\Rightarrow x = -1.2 - 1.7$

v=0 at $x = -0.5$

$\Rightarrow \boxed{A = .5}$

Time Required: Time to travel from negative extreme to point 2m below ceiling (⇒ point where $x = 2 - 1.7 = .3$)

Use $x = A\cos\omega t$.

$.3 = -.5\cos\sqrt{14}t$.

$-.6 = \cos\sqrt{14}t$

$2.214 = \sqrt{14}t$

$\boxed{0.59_{sec} = t}$

(Alternative SEE BOARD).