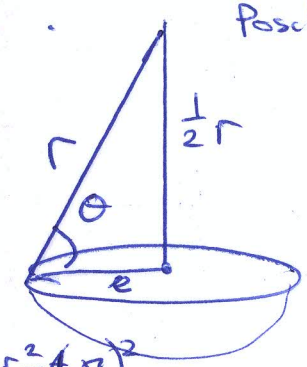
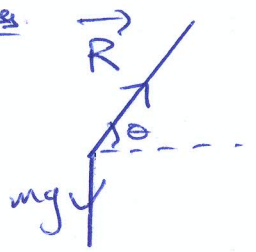
# Circular Motion (979) Honours

6(a)

Position




Forces




$\vec{R} = R\cos\theta\,\vec{i} + R\sin\theta\,\vec{j}$.

$\vec{R} = \frac{\sqrt{3}}{2}R\,\vec{i} + \frac{1}{2}R\,\vec{j}$.

$\vec{w} = -mg\,\vec{j}$.

$l^2 = r^2 - (\frac{1}{2}r)^2$

$l^2 = \frac{3}{4}r^2$

$\Rightarrow l = \frac{\sqrt{3}}{2}r$

$\Rightarrow \cos\theta = \frac{\sqrt{3}}{2}$, $\sin\theta = \frac{1}{2}$

Moving in horizontal circular path $\Rightarrow$

$\vec{j}$ forces $= 0 \Rightarrow R\sin\theta = mg$

$\Rightarrow R\frac{1}{2} = mg$

$\Rightarrow \boxed{\frac{R}{m} = 2g}$.

Circular path $\Rightarrow$

$\vec{i}$ forces $= m\frac{v^2}{l} \Rightarrow \frac{\sqrt{3}}{2}R = m\frac{v^2}{l}$

$\Rightarrow \frac{\sqrt{3}}{2}l\frac{R}{m} = v^2$

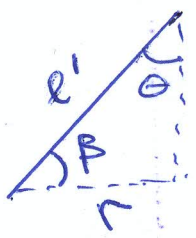
$\Rightarrow v^2 = \frac{\sqrt{3}}{2}\frac{\sqrt{3}}{2}r(2g)$

$\Rightarrow v^2 = \frac{2(\sqrt{3})^2}{4}rg$

$\Rightarrow v = \sqrt{\frac{6rg}{4}} = \frac{1}{2}\sqrt{6rg}$ qed.

6(b)

Position




$\cos\beta = \frac{r}{l'}$

Forces


$T = k(l'-l)$ by Hookes law

$\vec{T} = T\cos\beta\,\vec{i} + T\sin\beta\,\vec{j}$.

$\vec{i}$ cpts $\Rightarrow T\cos\beta = m\omega^2 r$

$\Rightarrow T = m\omega^2\frac{r}{\cos\beta}$

$\Rightarrow T = m\omega^2 l'$

Hooke $\Rightarrow \boxed{k(l'-l) = m\omega^2 l'}$ — ① qed.

Next show $\cos\theta = \frac{g}{l\omega^2} - \frac{mg}{kl}$.

$\vec{j}$ cpts $\Rightarrow T\sin\beta = mg$

$\Rightarrow \sin\beta = \frac{mg}{T}$

$\Rightarrow \cos\theta = \frac{mg}{T}$

Final expression has no $l'$ so we want to use $k(l'-l) = T$ with $l'$ substituted for.

① $\Rightarrow l' = \frac{kl}{k-m\omega^2} \Rightarrow T = k\left(\frac{kl}{k-m\omega^2}\right) - kl$

$\Rightarrow T = \frac{+k^2l - k^2l + klm\omega^2}{k-m\omega^2}$

$= \frac{klm\omega^2}{k-m\omega^2}$

$\therefore \cos\theta = \frac{mg}{\frac{klm\omega^2}{k-m\omega^2}} = \frac{mg}{klm\omega^2}[k-m\omega^2] = \frac{g}{l\omega^2} - \frac{mg}{kl}$