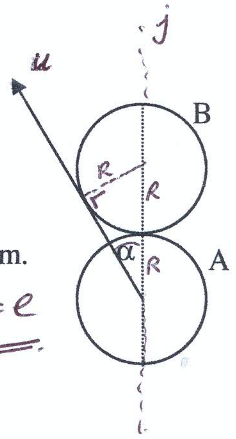
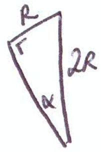
2009 HL

5 (b) A smooth sphere A, of mass m kg, moving with speed u, collides with a stationary identical smooth sphere B.
The direction of motion of A, before impact, makes an angle α with the line of centres at impact and just touches sphere B, as shown in the diagram.

The coefficient of restitution between the spheres is $\frac{4}{5}$. $= e$





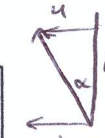
$\sin\alpha = \frac{R}{2R} = \frac{1}{2}$.

(i) Show that $\alpha = 30°$.

(ii) Find the direction in which each sphere travels after the collision.

Collision only in $\vec{j}$ dir $\Rightarrow$ $\vec{i}$ velocity remains unchanged.

(iii) Find the percentage loss in kinetic energy due to the col lision.



$u\cos\alpha\,\vec{j} = \frac{\sqrt{3}}{2}u\,\vec{j}$.

$-u\sin\alpha\,\vec{i} = -\frac{u}{2}\vec{i}$

(i) $\sin\alpha = \dfrac{r}{2r} \Rightarrow \alpha = 30°$ (see diagram) **5**

	Before		After
A	$-\frac{u}{2}\vec{i}+\frac{\sqrt{3}}{2}u\vec{j}$	M	$-\frac{u}{2}\vec{i}+v_1\vec{j}$
B	$0\vec{i}+0\vec{j}$	M	$0\vec{i}+v_2\vec{j}$

(ii) PCM in $\vec{j}$ dir. $\quad m\left(\dfrac{u\sqrt{3}}{2}\right) + m(0) = mv_1 + mv_2$ ①

NEL in $\vec{j}$ dir. $\quad v_1 - v_2 = -\dfrac{4}{5}\left(\dfrac{u\sqrt{3}}{2} - 0\right)$ ② **5**

$\div m$ ① $\frac{u\sqrt{3}}{2} = v_1 + v_2$

② $-\frac{2\sqrt{3}u}{5} = v_1 - v_2$

$u\left(\frac{\sqrt{3}}{2} - \frac{2\sqrt{3}}{5}\right) = 2v_1$

$\frac{u\sqrt{3}}{20} = v_1$

Sub into ①

$\frac{u\sqrt{3}}{20} + v_2 = \frac{u\sqrt{3}}{2}$

$v_2 = \frac{10u\sqrt{3} - u\sqrt{3}}{20}$

$= \frac{9u\sqrt{3}}{20}$

$\Rightarrow v_1 = \dfrac{u\sqrt{3}}{20}$ and $v_2 = \dfrac{9u\sqrt{3}}{20}$

velocity of A $= -\dfrac{u}{2}\vec{i} + \dfrac{u\sqrt{3}}{20}\vec{j}$

direction of A $= \tan^{-1}\left(\dfrac{\sqrt{3}}{10}\right)$ $\quad \tan^{-1}\left(\dfrac{\frac{u\sqrt{3}}{20}}{\frac{u}{2}}\right) = \tan^{-1}\left(\dfrac{\sqrt{3}}{10}\right)$

velocity of B $= 0\vec{i} + \dfrac{9u\sqrt{3}}{20}\vec{j}$ $\quad = \tan^{-1}\left(\dfrac{\frac{9u\sqrt{3}}{20}}{0}\right)$

direction of B = along line of centres **5**

undefined $\Rightarrow$ vertically.

(iii) KE before $= \frac{1}{2}mu^2 + \frac{1}{2}M(0)^2$

$\vec{u} = -\frac{u}{2}\vec{i} + \frac{\sqrt{3}}{2}u\vec{j}$

$|\vec{u}| = \sqrt{(-u/2)^2 + (\sqrt{3}/2\,u)^2}$

$= \sqrt{\frac{u}{4} + \frac{3}{4}u}$

$= \sqrt{u}$.

KE after $= \frac{1}{2}m\left\{\dfrac{u^2}{4} + \dfrac{3u^2}{400} + \dfrac{243u^2}{400}\right\} = \frac{1}{2}M\frac{346u^2}{400}$

KE lost $= \frac{27}{400}mu^2$

$\left|-\frac{u}{2}\vec{i} + \frac{u\sqrt{3}}{20}\vec{j}\right| = \sqrt{\left(\frac{u^2}{4}\right) + \frac{u^2 3}{400}} = \sqrt{\frac{103}{400}}u^2$

% KE lost $= \dfrac{\frac{27}{400}mu^2}{\frac{1}{2}mu^2} \times 100 = 13.5\%$ **5** **20**

$\left|0\vec{i} + \frac{9u\sqrt{3}}{20}\vec{j}\right| = \sqrt{0^2 + \frac{243u^2}{400}}$.