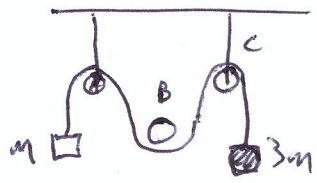
1992 HLC [Multibody Systems] A, C fixed



(i)

Masses (not fixed)	m	M (B)	$3m$
Accel (Assume)	$a\downarrow$	$\uparrow \frac{f+a}{2}$	$f\downarrow$
Forces:	$T\uparrow$, $mg\downarrow$	$T\uparrow$ $T\uparrow$, $Mg\downarrow$	$T\uparrow$, $3mg\downarrow$
N II:	$mg-T=ma$ (1)	$2T-Mg=M\left(\frac{f+a}{2}\right)$ (2)	$3mg-T=3mf$ (3)

(ii) This time we want to eliminate f and a from the equation so that we can find expression involving only T.

(1) $\Rightarrow a=\frac{1}{m}(mg-T)$

(3) $\Rightarrow f=\frac{1}{3m}(3mg-T)$

$\therefore$ (2) $\Rightarrow$ $2T-Mg=\frac{M}{2}\left[\frac{1}{3m}(3mg-T)+\frac{1}{m}(mg-T)\right]$

$\Rightarrow 2T-Mg=\frac{M}{2}\left[g-\frac{T}{3m}+g-\frac{T}{m}\right]$

$\Rightarrow 2T-Mg=\frac{Mg}{2}-\frac{TM}{6m}+\frac{Mg}{2}-\frac{TM}{2m}$

Everything with T to one side. $\Rightarrow 2T+\frac{TM}{6m}+\frac{TM}{2m}=Mg+Mg$

$\Rightarrow 2T+\frac{TM+3TM}{6m}=2Mg$

$\Rightarrow 2T+\frac{4TM}{6m}=2Mg$

$\Rightarrow 2T+\frac{2TM}{3m}=2Mg$

$\div 2$ $\Rightarrow T+\frac{TM}{3m}=Mg$ [LOOK AT ANSWER!]

$\Rightarrow T\left[1+\frac{M}{3m}\right]=Mg$

Divide by M $\Rightarrow$ $T\left[\frac{1}{M}+\frac{1}{3m}\right]=g$. (4) qed. (PHEW!)

(iii) Show if $M=3m$ B will remain at rest (ie $\Sigma f=0$ ie accel $=0$)

$M=3m$ $\therefore$ (4) $T\left[\frac{1}{3m}+\frac{1}{3m}\right]=g \Rightarrow T\left(\frac{2}{3m}\right)=g \Rightarrow T=\frac{3mg}{2}$.

$\therefore$ Nett forces on B $\Sigma f=2T-3mg$

$\Rightarrow \Sigma f=2\left(\frac{3mg}{2}\right)-3mg=3mg-3mg=0$.

$\therefore$ Nett accel of B $=0$, so it remains at Rest.