2004 – Projectiles Question

3. **(a)** A particle is projected from a point on the horizontal floor of a tunnel with maximum height of 8 m. The particle is projected with an initial speed of 20 m/s inclined at an angle α to the horizontal floor.

Find, to the nearest metre, the greatest range which can be attained in the tunnel.

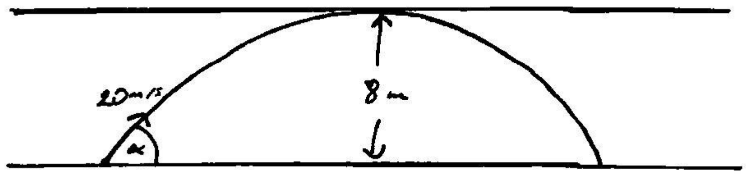
(b) A particle is projected up an inclined plane with initial velocity u m/s. The line of projection makes an angle α **with the horizontal** and the inclined plane makes an angle θ with the horizontal. (The plane of projection is vertical and contains the line of greatest slope.)

If the particle strikes the inclined plane at right angles, show that

$$\tan\alpha = \frac{1 + 2\tan^2\theta}{\tan\theta}.$$

2004

Q3/ (a)



AT MAX HEIGHT, $V_y = 0$ m/s, $S_y = 8$ m

$U_y = 20\sin\alpha$
$V_y = 0$
$a = -g$
$S_y = 8$
$t = ?$

$V^2 = u^2 + 2as$

$(0)^2 = (20\sin\alpha)^2 + 2(-g)(8)$

$0 = 400\sin^2\alpha - 16g$

$\frac{16g}{400} = \sin^2\alpha$

so,

$\sin\alpha = \frac{4\sqrt{g}}{20} \Rightarrow \boxed{\sin\alpha = \frac{\sqrt{g}}{5}}$

$V = u + at$

$0 = 20\sin\alpha - gt$

$t = \frac{20\sin\alpha}{g} \Rightarrow t = \frac{20\left(\frac{\sqrt{g}}{5}\right)}{g} \Rightarrow t = \frac{4\sqrt{g}}{g} \Rightarrow \boxed{t = \frac{4}{\sqrt{g}}}$

so, TIME TO FULL RANGE IS $2\left(\frac{4}{\sqrt{g}}\right) = \frac{8}{\sqrt{g}}$

$U_x = 20\cos\alpha$
$V_x = -$
$a_x = 0$
$S_x = ?$
$t = \frac{8}{\sqrt{g}}$

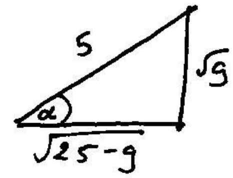
$S = ut + \frac{1}{2}at^2$

$S = 20\cos\alpha\left[\frac{8}{\sqrt{g}}\right] + \cancel{\frac{1}{2}(0)t^2}$

$S_x = \frac{160\cos\alpha}{\sqrt{g}}$

2006

Q3/ But : $\sin\alpha = \dfrac{\sqrt{g}}{5}$



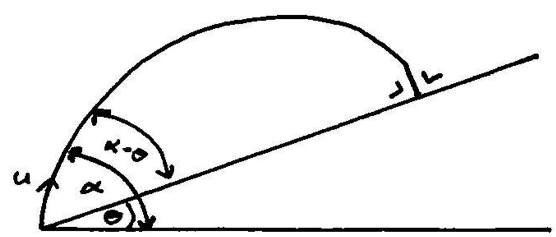
so, $\cos\alpha = \dfrac{\sqrt{25-g}}{5}$

so,

$$S_x = \frac{160\left[\frac{\sqrt{25-g}}{5}\right]}{\sqrt{g}} \Rightarrow \frac{32\sqrt{25-g}}{\sqrt{g}}$$

$$\Rightarrow \frac{32\sqrt{25-9.8}}{\sqrt{9.8}} = 39.7 \approx \underline{\underline{40\,m}} = \text{Max Range}$$

(b)



LANDS AT RIGHT ANGLES SO, $V_x = 0$ WHEN $S_y = 0$

FIND t WHEN $S_y = 0$

$U_y = u\sin(\alpha-\theta)$ $\qquad$ $s = ut + \frac{1}{2}at^2$

$V_y = -$

$a_y = -g\cos\theta$ $\qquad$ $0 = [u\sin(\alpha-\theta)]t + \frac{1}{2}(-g\cos\theta)t^2 \quad (\div t)$

$S_y = 0$

$t = ?$ $\qquad$ $0 = u\sin(\alpha-\theta) - \frac{g\cos\theta}{2}t$

$$\boxed{t = \frac{2u\sin(\alpha-\theta)}{g\cos\theta}}$$

2006

Q3/ (b) At this time, $V_x = 0$

$$u_x = u\cos(\alpha-\theta)$$
$$V_x = 0$$
$$a_x = -g\sin\theta$$
$$S_x = -$$
$$t = \frac{2u\sin(\alpha-\theta)}{g\cos\theta}$$

$$v = u + at$$

$$0 = u\cos(\alpha-\theta) - g\sin\theta\left[\frac{2u\sin(\alpha-\theta)}{g\cos\theta}\right]$$

$$\Rightarrow\ 0 = u\cos(\alpha-\theta) - \frac{2u\sin\theta\sin(\alpha-\theta)}{\cos\theta}$$

$$\Rightarrow\ \frac{2\sin\theta\sin(\alpha-\theta)}{\cos\theta} = \cos(\alpha-\theta)$$

$$\frac{2\sin\theta\sin(\alpha-\theta)}{\cos\theta\cos(\alpha-\theta)} = 1$$

$$2\tan\theta\tan(\alpha-\theta) = 1$$

$*\left[\text{From tables:}\quad \tan(A-B) = \frac{\tan A - \tan B}{1+\tan A\tan B}\right]$

so,

$$2\tan\theta\left[\frac{\tan\alpha - \tan\theta}{1+\tan\alpha\tan\theta}\right] = 1$$

$$2\tan\theta\tan\alpha - 2\tan^2\theta = 1 + \tan\alpha\tan\theta$$

$$2\tan\theta\tan\alpha - \tan\theta\tan\alpha = 1 + 2\tan^2\theta$$

$$\tan\theta\tan\alpha = 1 + 2\tan^2\theta$$

so,

$$\tan\alpha = \frac{1+2\tan^2\theta}{\tan\theta}$$

Q.