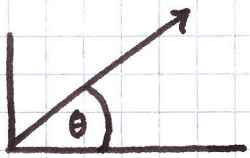
2003 Question 3.

a.



$u_x = u\cos\theta$ $\quad u_y = u\sin\theta$

$v_x = u\cos\theta$ $\quad v_y = u\sin\theta - gt$

$s_x = u\cos\theta t$ $\quad s_y = u\sin\theta t - \frac{1}{2}gt^2$

at maximum range, $s_y = 0 \Rightarrow \quad u\sin\theta t - \frac{1}{2}gt^2 = 0$

$t(2u\sin\theta - gt) = 0$

$t = \frac{2u\sin\theta}{g}$

Range is $s_x = u\cos\theta t$

$= u\cos\theta\left(\frac{2u\sin\theta}{g}\right) = \frac{2u^2\cos\theta\sin\theta}{g}$ Range

At maximum height, $v_y = 0 \Rightarrow \quad u\sin\theta - gt = 0$

$t = \frac{u\sin\theta}{g}$

Max height is $s_y = u\sin\theta t - \frac{1}{2}gt^2$

$s_y = u\sin\theta\left(\frac{u\sin\theta}{g}\right) - \frac{1}{2}g\left(\frac{u\sin\theta}{g}\right)^2$

$= \frac{u^2\sin^2\theta}{g} - \frac{u^2\sin^2\theta}{2g} = \frac{u^2\sin^2\theta}{2g}$ Height

Range = 5x max height

$\frac{2u^2\cos\theta\sin\theta}{g} = \frac{5u^2\sin^2\theta}{2g}$

$4\cos\theta\sin\theta = 5\sin^2\theta$

$\tan\theta = \frac{4}{5}$ $\quad \theta = 38.7°$