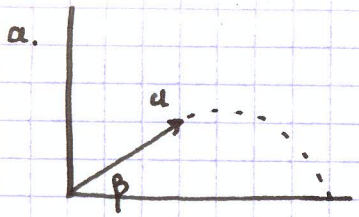
2000 Question 3.

a.



$u_x = u\cos\beta$ $\quad u_y = u\sin\beta$

$V_x = u\cos\beta$

$S_x = u\cos\beta t$ $\quad S_y = u\sin\beta t - \frac{1}{2}gt^2$

hits ground $\Rightarrow$ $0 = u\sin\beta t - \frac{1}{2}gt^2$

$0 = u\sin\beta - \frac{gt}{2}$

$\frac{2u\sin\beta}{g} = t$ ← sub

$S_x = u\cos\beta\left(\frac{2u\sin\beta}{g}\right)$

$2\sin A\cos A = \sin 2A$ $\quad S_x = \frac{u^2 2\sin\beta\cos\beta}{g}$

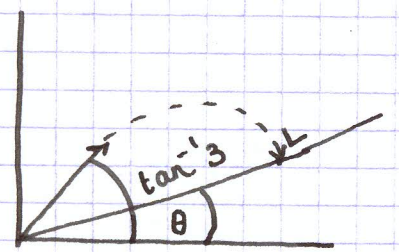
$$\boxed{S_x = \frac{u^2\sin 2\beta}{g} \quad \text{Q.E.D.}}$$

S_x maximum: $\sin 90 = 1$ $\quad \sin < 90 = < 1$

$\Rightarrow$ max $\sin 2\beta = 1$ $\Rightarrow$ $2\beta = 90°$

$\beta = 45°$

b.



at R, $v_x = 0$, $s_y = 0$

$u_x = u\cos(\alpha-\theta)$ $\quad u_y = u\sin(\alpha-\theta)$

$g_x = -g\sin\theta$ $\quad g_y = -g\cos\theta$

$V_x = u\cos(\alpha-\theta) - g\sin\theta t$

$S_y = u\sin(\alpha-\theta)t - \frac{1}{2}g\cos\theta t^2$

$V_x = 0$: $\quad 0 = u\cos(\alpha-\theta) - g\sin\theta t$

$\frac{u\cos(\alpha-\theta)}{g\sin\theta} = t$ ← sub

$0 = u\sin(\alpha-\theta)\left[\frac{u\cos(\alpha-\theta)}{g\sin\theta}\right] - \frac{1}{2}g\cos\theta\left[\frac{u\cos(\alpha-\theta)}{g\sin\theta}\right]^2$

$0 = \frac{\cos(\alpha-\theta)u^2\left[2\sin(\alpha-\theta)\sin\theta - \cos\theta\cos(\alpha-\theta)\right]}{2g\sin^2\theta}$

÷ across by $\cos(\alpha-\theta)\cos\theta$

$0 = \frac{2\sin(\alpha-\theta)\sin\theta}{\cos(\alpha-\theta)\cos\theta} - \frac{\cos\theta\cos(\alpha-\theta)}{\cos\theta\cos(\alpha-\theta)}$

$\tan(A+B) = \frac{\tan A+\tan B}{1-\tan A\tan B}$

and $\tan(-A) = -\tan A$

$0 = 2\tan(\alpha-\theta)\tan\theta - 1$

$1 = 2\tan\theta\left[\frac{\tan\alpha - \tan\theta}{1+\tan\alpha\tan\theta}\right]$

$1 = 2\tan\theta\left[\frac{3-\tan\theta}{1+3\tan\theta}\right]$

$1+3\tan\theta = 6\tan\theta - 2\tan^2\theta$

$2\tan^2\theta - 3\tan\theta + 1 = 0$

$(2\tan\theta - 1)(\tan\theta - 1) = 0$

$\Rightarrow \tan\theta = \frac{1}{2}$ or $\tan\theta = 1$

$\Rightarrow \theta = 26°57'$ or $45°$