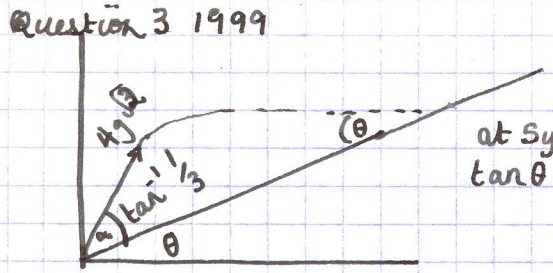
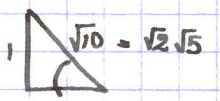
Question 3 1999



at $S_y = 0$

$\tan\theta = \frac{-V_y}{V_x}$



$\cos\alpha = \frac{3}{\sqrt{2}\sqrt{5}}$

$\sin\alpha = \frac{1}{\sqrt{2}\sqrt{5}}$

$u_x = u\cos\alpha = 4g\sqrt{2}\cdot\frac{3}{\sqrt{2}\sqrt{5}} = \frac{12g}{\sqrt{5}}$

$u_y = u\sin\alpha = 4g\sqrt{2}\cdot\frac{1}{\sqrt{2}\sqrt{5}} = \frac{4g}{\sqrt{5}}$

$g_x = -g\sin\theta$ $\quad g_y = -g\cos\theta$

$V_x = \frac{12g}{\sqrt{5}} - g\sin\theta t$ $\quad V_y = \frac{4g}{\sqrt{5}} - g\cos\theta t$

$S_y = \frac{4g}{\sqrt{5}}t - \frac{1}{2}g\cos\theta t^2$

$0 = \frac{4g}{\sqrt{5}} - \frac{g}{2}\cos\theta t$

$\boxed{\frac{8g}{\sqrt{5}\cos\theta} = t}$

sub into V_x: $V_x = \frac{12g}{\sqrt{5}} - g\sin\theta\left[\frac{8}{\sqrt{5}\cos\theta}\right]$

sub into V_y: $V_y = \frac{4g}{\sqrt{5}} - g\cos\theta\left[\frac{8}{\sqrt{5}\cos\theta}\right]$

$V_x = \frac{12g - 8g\tan\theta}{\sqrt{5}}$ $\quad V_y = \frac{4g - 8g}{\sqrt{5}} = -\frac{4g}{\sqrt{5}}$

$\Rightarrow -\frac{V_y}{V_x} = \frac{4g}{12g - 8g\tan\theta}$

$\tan\theta = \frac{4g}{12g - 8g\tan\theta}$

$12\tan\theta - 8\tan^2\theta = 4$

$0 = 2\tan^2\theta - 3\tan\theta + 1$

$(2\tan\theta - 1)(\tan\theta - 1) = 0$

$\tan\theta = \frac{1}{2}$ or 1

$\theta = 26°34'$ or $45°$

b. $\tan\theta = 0.5$

$V_x = \frac{12g - 4g}{\sqrt{5}} = \frac{8g}{\sqrt{5}}$

$V_y = -\frac{4g}{\sqrt{5}}$

i. magnitude of velocity $= \vec{v} = \sqrt{V_x^2 + V_y^2}$

$= \sqrt{\left(\frac{8g}{\sqrt{5}}\right)^2 + \left(\frac{-4g}{\sqrt{5}}\right)^2}$

$= \frac{\sqrt{80}}{\sqrt{5}}g$ $\quad \boxed{= 4g}$

ii. find total energy at p = total energy at q

$KE = \frac{1}{2}m\vec{v}^2$

$PE = mg\bar{h}$

where $\bar{h} = S_x\sin\theta$ →

$S_x = \frac{12g}{\sqrt{5}}\left(\frac{8}{\sqrt{5}}\cdot\frac{\sqrt{5}}{2}\right) - \frac{1}{2}g\frac{1}{\sqrt{5}}\left(\frac{8}{\sqrt{5}}\cdot\frac{\sqrt{5}}{2}\right)^2$

$= \frac{48g}{\sqrt{5}} - \frac{8g}{\sqrt{5}}$

at p: $KE = \frac{1}{2}m\left(\frac{4g\sqrt{2}}{\sqrt{2}}\right)^2 = 16mg^2$

at q: $KE = \frac{1}{2}m(4g)^2 = 8mg^2$

$PE = mg\left(\frac{48g}{\sqrt{5}} - \frac{8g}{\sqrt{5}}\right)\frac{1}{\sqrt{5}}$

$= 8mg^2$

$\Rightarrow E$ at p $= E$ at q.