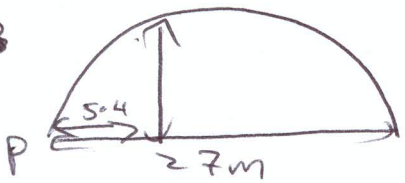
Proj 95 Q3



$T = 3$ secs

$\vec{u} = u\cos\alpha\,\vec{i} + u\sin\alpha\,\vec{j}$

$\vec{g} = 0\vec{i} - g\vec{j}$.

$\vec{r}(t) = u\cos\alpha\, t\,\vec{i} + \left(u\sin\alpha\, t - \frac{g}{2}t^2\right)\vec{j}$.

At $t = 3$: $\vec{r}(t)\cdot\vec{i} = 27$

$\Rightarrow u\cos\alpha(3) = 27$

$\Rightarrow u\cos\alpha = 9$

Also $\left(\vec{r}(3)\right)\cdot\vec{j} = 0$

$u\sin\alpha(3) - \frac{g}{2}(3)^2 = 0$

$\Rightarrow (u\sin\alpha)3 = \frac{9(9.8)}{2}$

$\Rightarrow u\sin\alpha = 14.7$

$\therefore \vec{u} = 9\vec{i} + 14.7\vec{j}$.

(ii)

$\vec{r}(t) = 9t\,\vec{i} + \left(14.7t - 4.9t^2\right)\vec{j}$

$\vec{v}(t) = 9\,\vec{i} + (14.7 - 9.8t)\vec{j}$.

height of wall

Find t where $\left(\vec{r}(t)\right)\cdot\vec{i} = 5.4$

$9t = 5.4$

$t = 0.6$

$\Rightarrow h = \left(\vec{r}(0.6)\right)\cdot\vec{j}$

$\Rightarrow h = 14.7(0.6) - 4.9(0.6)^2$

$\Rightarrow h = 8.82 - 1.764$

$\Rightarrow \boxed{h = 7.056\text{m}}$

(iii) $\vec{v}(0.6) = 9\vec{i} + (14.7 - 9.8(0.6))\vec{j}$

$\vec{v}(0.6) = 9\vec{i} + 8.82\vec{j}$

$\rightarrow$ Speed $= |\vec{v}(0.6)|$

$= \sqrt{9^2 + (8.82)^2}$

$= 12.6$ m/s.

(i) 20

Stone at right angles $\Rightarrow$

$\vec{v}(T)\cdot\vec{i} = 0$

(i) $\vec{u} = 20\cos\theta\,\vec{i} + 20\sin\theta\,\vec{j}$

$\vec{g} = -g\sin 30\,\vec{i} - g\cos 30\,\vec{j}$.

$\vec{r}(t) = \left(20\cos\theta\, t - \frac{g}{4}t^2\right)\vec{i} + \left(20\sin\theta\, t - \frac{g\sqrt{3}}{4}t^2\right)\vec{j}$

$\vec{v}(t) = \left(20\cos\theta - \frac{g}{2}t\right)\vec{i} + \left(20\sin\theta - \frac{g\sqrt{3}}{2}t\right)\vec{j}$

Find expression for T:

$\left(\vec{r}(T)\right)\cdot\vec{j} = 0$

$20\sin\theta\, t - \frac{g\sqrt{3}}{4}t^2 = 0$

$\Rightarrow t = 0$ OR $T = \frac{80\sin\theta}{g\sqrt{3}}$.

To stone at right angles:

$\left(\vec{v}(T)\right)\cdot\vec{i} = 0 \Rightarrow 20\cos\theta - \frac{g}{2}\left(\frac{80\sin\theta}{g\sqrt{3}}\right) = 0$

$\Rightarrow 20\cos\theta = \frac{40\sin\theta}{\sqrt{3}}$

$\Rightarrow \frac{\sqrt{3}}{2} = \tan\theta$.

$\Rightarrow \theta = 40.89°$.

(ii) $\theta = 45°$, then $T = \frac{80\sin 45}{g\sqrt{3}} = \frac{80}{g\sqrt{6}}$

Calculate $\tan\alpha = \frac{|\vec{v}(T)\cdot\vec{j}|}{(\vec{v}(T)\cdot\vec{i})}$

$\vec{v}(T) = \left(20\cos 45 - \frac{g}{2}\left(\frac{80}{g\sqrt{6}}\right)\right)\vec{i} + \left(20\sin 45 - \frac{g\sqrt{3}}{2}\left(\frac{80}{g\sqrt{6}}\right)\right)\vec{j}$.

$= \left(\frac{20}{\sqrt{2}} - \frac{40}{\sqrt{6}}\right)\vec{i} + \left(\frac{20}{\sqrt{2}} - \frac{40}{\sqrt{2}}\right)\vec{j}$.

$= (-2.18)\vec{i} + (-14.142)\vec{j}$.

$\therefore \tan\alpha = \frac{|-14.142|}{-2.18} = -6.48$

$\therefore \alpha$ of base $\therefore$ bounces back down the plane.