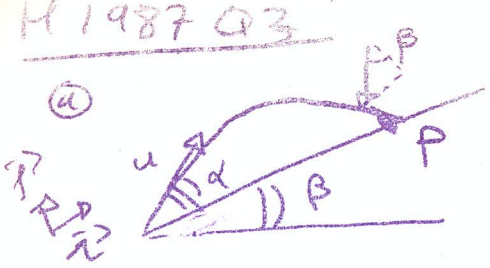
H 1987 Q3




(a)

$\tan\beta = \frac{1}{4} \Rightarrow \sin\beta = \frac{1}{\sqrt{17}} \quad \cos\beta = \frac{4}{\sqrt{17}}$

$\vec{u} = u\cos\alpha\,\vec{i} + u\sin\alpha\,\vec{j}$

$\vec{g} = -g\sin\beta\,\vec{i} - g\cos\beta\,\vec{j}$

$\vec{g} = -\frac{g}{\sqrt{17}}\vec{i} - \frac{4g}{\sqrt{17}}\vec{j}$.

$\vec{v}(t) = \left(u\cos\alpha - \frac{g}{\sqrt{17}}t\right)\vec{i} + \left(u\sin\alpha - \frac{4g}{\sqrt{17}}t\right)\vec{j}$. 5 marks.

$\vec{r}(t) = \left(u\cos\alpha\, t - \frac{g}{2\sqrt{17}}t^2\right)\vec{i} + \left(u\sin\alpha\, t - \frac{2g}{\sqrt{17}}t^2\right)\vec{j}$. 5 marks

At P (Theory): $(r(t))_{\vec{j}} = 0 \Rightarrow u\sin\alpha\, t - \frac{2g}{\sqrt{17}}t^2 = 0$

$\Rightarrow t = 0$ or $t = \frac{\sqrt{17}\,u\sin\alpha}{2g}$ 5 marks

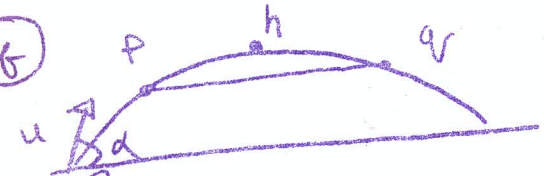
At P (general): $(v)_{\vec{i}} = 0 \Rightarrow u\cos\alpha - \frac{g}{\sqrt{17}}t = 0$

$\Rightarrow \Rightarrow T = \frac{\sqrt{17}\,u\cos\alpha}{g}$ 5 marks.

$\Rightarrow \frac{\sqrt{17}\,u\sin\alpha}{2g} = \frac{\sqrt{17}\,u\cos\alpha}{g}$

$\Rightarrow \frac{\sin\alpha}{\cos\alpha} = 2 \Rightarrow \boxed{\tan\alpha = 2}$ 5 marks.

(b)

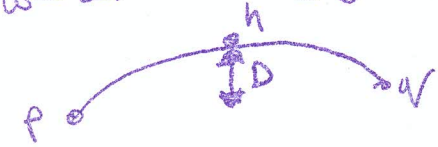



different α:

The particle will take 4 secs to travel from P to Q. 5 marks

It will take 2 secs to travel from h to Q where h is the highest point. 5 marks

So we are really just considering the motion P to Q where the time to go from P to h or h to Q is 2 seconds.




Consider the j cpt of the motion from h to Q [with h = 0]

$\Rightarrow D = 0t + \frac{1}{2}gt^2$ 5 marks

$= \frac{1}{2}\cdot 9.8\cdot 2^2$ 5 marks

$= 19.6$m q.e.d. 5 marks

OR:

$\vec{r}(t) = u\cos\alpha\, t\,\vec{i} + (u\sin\alpha\, t - \frac{1}{2}gt^2)\vec{j}$

$\vec{v}(t) = u\cos\alpha\,\vec{i} + (u\sin\alpha - gt)\vec{j}$

$D = H - (\vec{r}(t^* - 2))_{\vec{j}} = (\vec{r}(t^*))_{\vec{j}} - (\vec{r}(t^* - 2))_{\vec{j}}$ where $t^*$ = time to reach max height

And $t^*$: $(v)_{\vec{j}} = 0 \Rightarrow \boxed{t^* = \frac{u\sin\alpha}{g}} \Rightarrow H = u\sin\alpha\left[\frac{u\sin\alpha}{g}\right] - \frac{g}{2}\left(\frac{u\sin\alpha}{g}\right)^2$

$= \frac{u^2\sin^2\alpha}{2g}$

$(\vec{r}(t^* - 2))_{\vec{j}} = u\sin\alpha\left(\frac{u\sin\alpha}{g} - 2\right) - \frac{g}{2}\left(\frac{u\sin\alpha}{g} - 2\right)^2 = \frac{u^2\sin^2\alpha}{g} - 2u\sin\alpha - \frac{g}{2}\left(\frac{u^2\sin^2\alpha}{g^2} - \frac{4u\sin\alpha}{g} + 4\right)$

FINISH OFF AS ABOVE