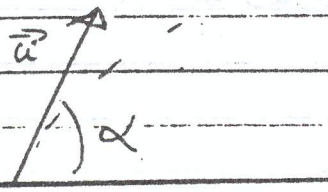
1986 Q3



Max height = total Range Show $\tan\alpha = 4$

$\vec{u} = u\cos\alpha\,\vec{i} + u\sin\alpha\,\vec{j}$

$\vec{g} = -g\vec{j}$

$\vec{v}(t) = u\cos\alpha\,\vec{i} + (u\sin\alpha - gt)\vec{j}$ ... (a)

$\vec{r}(t) = u\cos\alpha\,t\,\vec{i} + (u\sin\alpha\,t - \frac{g}{2}t^2)\vec{j}$

Find Range

Just find the time for the Range

$(\vec{r})\cdot\vec{j} = 0 \Rightarrow u\sin\alpha\,t - \frac{1}{2}gt^2 = 0$

$\Rightarrow t = 0$ or $T = \frac{2u\sin\alpha}{g}$

$(\vec{r})\cdot\vec{i}$ = Range $= u\cos\alpha\left[\frac{2u\sin\alpha}{g}\right] = \frac{2u^2\sin\alpha\cos\alpha}{g}$

Find Max. Height:

First find time to reach max height

At max height $(\vec{v})\cdot\vec{j} = 0 \Rightarrow u\sin\alpha - gt = 0$

$\Rightarrow t = \frac{u\sin\alpha}{g}$

$\therefore$ Max height $(\vec{r}(t))\cdot\vec{j} = u\sin\alpha\left[\frac{u\sin\alpha}{g}\right] - \frac{1}{2}g\left[\frac{u\sin\alpha}{g}\right]^2$

$= \frac{u^2\sin^2\alpha}{g} - \frac{1}{2}\frac{u^2\sin^2\alpha}{g}$

$= \frac{1}{2}\left[\frac{u^2\sin^2\alpha}{g}\right]$

Now Range = max height.

$\frac{2u^2\cos\alpha\sin\alpha}{g} = \frac{1}{2}\frac{u^2\sin^2\alpha}{g}$

$\Rightarrow 2\cos\alpha\sin\alpha = \frac{1}{2}\sin^2\alpha$

$\Rightarrow 4 = \frac{\sin 2\alpha}{\cos\alpha\sin\alpha}$

$\Rightarrow \boxed{4 = \tan\alpha}$ qed.

$\tan\alpha = 4$.

$\Rightarrow \cos\alpha = \frac{1}{\sqrt{17}}, \sin\alpha = \frac{4}{\sqrt{17}}$